Incorporation of geological uncertainty in pit optimization with geostatistics simulation

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Abstract

Risk mapping processes in mine planning and ore recovery are constantly used in the mining industry to increase decision making certainty based on the available information. However, it is not possible to predict the risk behavior in all of the project's boundary conditions and small variations in some of these conditions can cause a great impact on its financial return. Among the countless uncertainties existing in a mining project (operational, costs, market change), many authors define the geological uncertainty as the most critical one, capable of influencing the success of the project. Measurement and evaluation of the geological uncertainty of a mine planning project is crucial because the calculated risk can be translated into a financial risk of the project. This article presents a possible way to consider the geological uncertainty in the pit optimization step by using sequential Gaussian simulation. The results obtained from the case study on a copper deposit results in a simple procedure with significant increase in reliability for the project.

Keywords: risks, geological uncertainty, sequential Gaussian simulation, mine planning, pit optimization.

1. Introduction

The long term mine planning, also called strategic mine planning, corresponds to the process of determining the best design and mining scheduling, according to a previously established strategy. It is considered a key element for the success of a mining enterprise, since it controls the decision making concerning their conduction and development. One of the main challenges for the realization of a mining plan is to ensure that the developed plan is feasible and considers the geological and operational variabilities of the mine. Some plans, which are developed without knowledge of the possible variables, have a lower reliability that can result in intuitive decisions making, which is not always optimal, and may harm the achievement of the goals set for a long term.

Quantifying the uncertainties and risks involved in a planning project is a step that has a large influence on the pit limits and production scheduling. Impacts on the order of millions of dollars can be provided by small variations and/or changes in project boundary conditions. Cost of capital, the market price of ore, attractive rates and operating costs are factors usually tested to analyze the feasibility of a project. However, even though these are considered by many authors as the main factor of failure in achieving the targets in mining industry, they are rarely considered as an uncertainty related to the geological model.

Usually the estimate and distribution of grades in a deposit are done by geostatistical and classical methods. Such methods result only in the estimated average values for the deposit, not being able to reproduce the actual spatial variability of data in situ. That variability is required in sensitivity engineering project analyses, since the variability of these contents imply variations in the final value of the project, whose impact is generally unknown.

The stochastic conditional simulation techniques allowed the real variability in situ to be evaluated. The methods of stochastic simulation were originally developed to correct the smoothing effect displayed on maps produced by kriging. Contrary to kriging, stochastic simulation methods do not result in a single estimate of the map of the variable of interest. The geostatistical simulation methods aim is to reproduce the variability in situ, and the spatial continuity of the original data, by the generation of equiprobable scenarios, conditioned to data, reproducing the statistical characteristics of 1st and 2nd order of sample data. That way, the degree of uncertainty associated with the estimates can be evaluated.

Herein, a comparison is presented of a pit obtained by the optimization of a block model of a copper deposit, which was estimated by ordinary kriging, with the pits obtained by simulated models to measure risks due to no consideration of the geological variability in the project.

2. Materials and methods

The kriging estimate is performed by minimizing the variance of the estimation error. Thus, it can be concluded that the estimate aims to offer the best possible estimate of an attribute without reproducing the original spatial heterogeneity of the data.

However, this minimization provides lower variance than the original data. This reduction in variability is known as smoothing. As cited by Goovaerts (1997), all interpolation algorithms tend to smooth the spatial variability of the attribute. This effect is characterized by underestimation of high values and

overestimation of low values. Smoothing can be easily observed to comparing the histogram of a database with the result of their kriging estimation.

As shown by Olea (1999), the simulation was the solution adopted to solve the smoothing problem of kriging. But, according to the author, the gain in overall accuracy causes the reduction of local accuracy. In fact, the realizations of the simulation scenarios are not free from errors of reproducing reality and, on average, the mistakes are higher than kriging.

Therefore, simulations should not be thought of as a substitute method for

kriging, but as a variability and uncertainty verification tool involved in kriging estimation, since kriging is still the best unbiased estimator.

Presented initially by Matheron (1973), the stochastic conditional simulation techniques allowed the variability and uncertainty involved in the estimation of mineral deposits to be quantitatively evaluated. Until that moment the kriging variance was the only existing way to evaluate the estimates. But Journel (1986) and, later, Brus and Gruijter (1993) began to question the use of this parameter as an estimative quality index.

2.1 Grade simulation

According to Deutsch and Journel (1992), the absence of an estimation error component $(R(x_0))$ provides smooth-

ing of kriging. The equation presented below shows that the real value of a particular attribute in a point x_0 can be

written as the sum of its estimate with the estimated error.

$$Z(x_0) = Z^*(x_0) + R(x_0) \qquad (1)$$

Thus, to have access to real variability of the random function $Z(x_0)$, the random function $R(x_0)$ should be

simulated many times and added to the estimated value. Each *ith* simulated value in each of the simulation scenarios can be

written as the sum of the kriging value with the *l-th* realization of the random function $R(x_0)$.

$$Z^l(x_0) = Z^*(x_0) + R^l(x_0) \qquad (2)$$

Existing simulation methods seek to randomly determine the error component based on the known method of Monte Carlo. Thus, as the process is random, the realizations will be different, but honoring the sample histogram and the sample variogram model.

Histogram and variogram reproduction is classified as an overall accuracy. For the result of interpolation, the same sample statistic used in the estimation was maintained. Actually kriging, classified as

a local precision method, does not produce the histogram and sample variogram, but shows a high correlation between estimated and used samples.

Among the many methods of stochastic simulation, the algorithms of sequential simulation methods are the most used to reproduce the spatial distribution and uncertainty of different variables (Soares, 2001, p. 911). This greater utilization results from a greater simplicity of execution and efficiency, since other

methods, besides having a more complex use, may have limitations and/or problems with their results.

The multiplicand presented in Equation 3 summarizes the theoretical basis of the sequential simulation methods, where each simulation generates a new point that is used to update the conditional cumulative distribution function. Thus, the conditional cumulative distribution function is always updated by a subsequent simulated value and the n sampling points.

$$
\begin{aligned}
F(x_1,\dots,x_N; Z_1,\dots,Z_N \mid (n)) &= F(x_1; Z_1 \mid (n)) \\
&\quad .F(x_2; Z_2 \mid (n+1)) \\
&\quad \dots \\
&\quad .F(x_{N-1}; Z_{N-1} \mid (n+N-2)) \\
&\quad .F(x_N; Z_N \mid (n+N-1)) \\
&= \prod_{i=1}^{N} F(x_i; Z_i \mid (n+i-1))
\end{aligned} \qquad (3)
$$

The main and most common method used of sequential simulation on deposit modeling is the sequential Gaussian simulation method (SGS) because of its simplicity, flexibility and reasonable efficiency (Deutsch, 2002, p. 162). To use this method, one should

work with a normal distribution with null mean and unit variance, so data must receive prior to processing.

The block model of the deposit, with a total of 33.807 blocks, was simulated in 50 scenarios using the sequential Gaussian simulation

method. The estimation method used by the simulation was simple kriging. In Figure 1, there can be seen three different scenarios of the same section of the block model plan where the warmer colors represent blocks with higher grades.
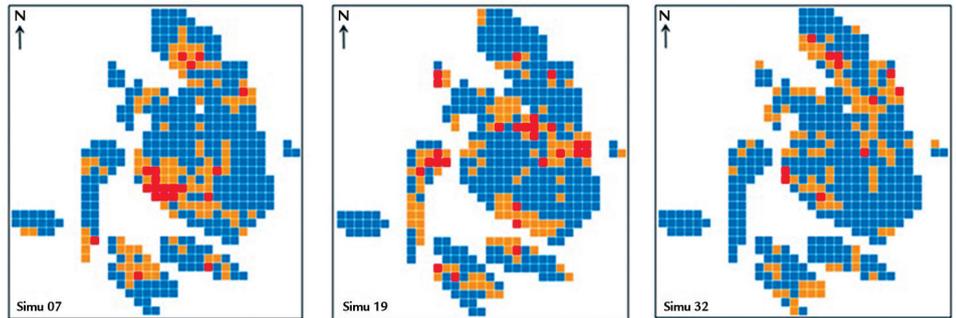
Figure 1
Plan view of scenarios 7, 19 and 32 obtained from the simulation.

After performing the simulations, it is important to verify that the scenarios obtained as a result satisfactorily reproduce the distribution of sample data and used variogram models. The 50 scenarios reproduce quite acceptably the sample distribution (histograms) and spatial continuity (variograms). The figure below is a variogram of all scenarios were well adherent to the variogram model in three main directions.
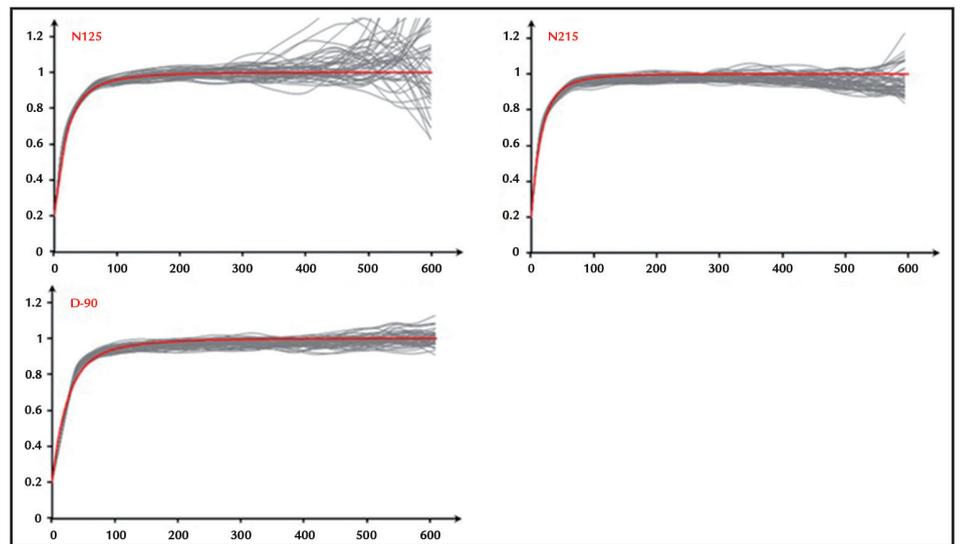

Figure 2
Validation of spatial continuity in the three main anisotropic directions.

2.2 Pit optimization

Obtaining the optimal pit was achieved by using the three dimensional optimization algorithm of Lerchs-Grossman (1965), by *NPV Scheduler* 4 software. In the following table are shown the values and parameters used. The sale price considered for copper was 2.06 $/lb (about 4,540 $/t).

Parameter	Value
Copper price	4,540.00 $/t
Metallurgical recovery	83.70 %
Mining cost	3.50 $/t$_{ROM}$
Processing cost	10.00 $/t$_{ROM}$
General angle	62°

Table 1
Technical and economic parameters considered in the optimization step.

Figure 3 presents the mathematical pit obtained by optimizing the model estimated by ordinary kriging.
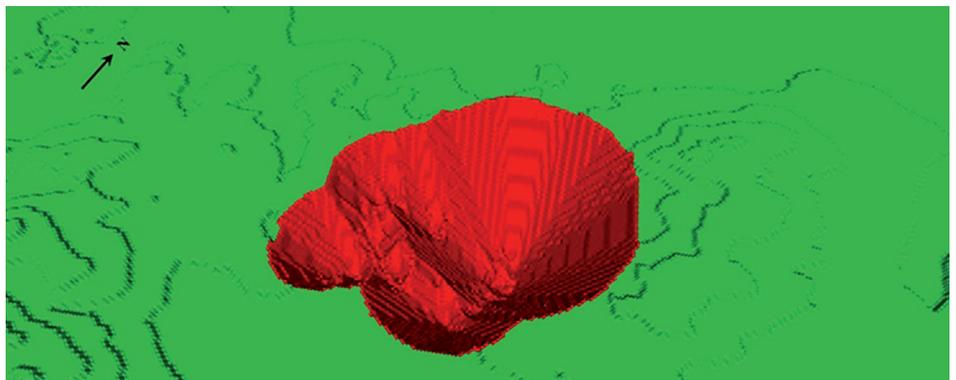

Figure 3
Perspective view of the pit obtained by mathematical optimization.

The optimization of simulated scenarios follows the same premises adopted for the kriging model (Table 1). The only difference is that instead of using the estimated copper value by ordinary kriging as product, the value obtained by simulation in each scenario was used.

Due to the great demand of time and the amount of information generated, not all scenarios were optimized. In order to maintain the representation of all the simulations without the need to calculate the 50 pits, a criterion was adopted to select the simulations that participated in the study. In this criterion, the optimal extraction sequence (OES) obtained by optimizing the kriging model was used to calculate the NPV of the 50 scenarios. Thus, the 50 NPV values found were ranked in increasing order and 11 scenarios were chosen to participate in the evaluation of the pit (Table 2), which divided the entire population into a range of 5 in 5 NPV values.

Scenario	SIM	Scenario	SIM	Scenario	SIM
1	39	5	1	9	25
2	13	6	18	10	35
3	34	7	9	11	45
4	43	8	50		

Table 2
Selected scenarios
to calculate the final pit.

3. Results

The figure 4 shows the limits in the plant of 11 pits for the selected scenarios compared to the pit limit of the kriging model (red line).
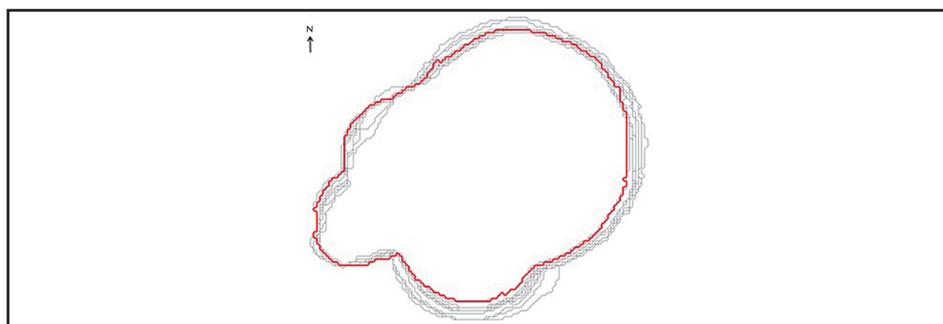

Figure 4
Comparison of the max
limit of the kriging model pit (in red) with the pits of simulated scenarios (in gray).

It can be seen that in certain regions the kriging model pit presents an optimistic behavior, being located externally to the simulated limits. While in other regions, the kriging pit has a more conservative behavior in relation to the simulations. Such behavior can also be viewed in depth as shown in the following Figure 5.

Areas with large variation limits in plant and depth can be used to define potential targets for additional drilling (Godoy, 2009). Thus, this great fluctuation limits results from a high variability provided by the absence or small amount of information.
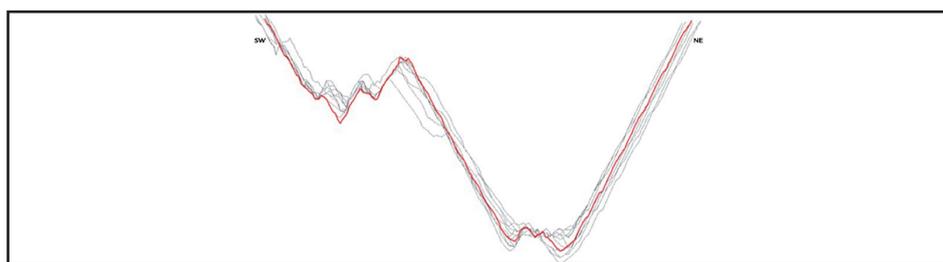

Figure 5
Vertical section in the direction
SW-NE presenting the difference in depth of the kriging model pit (in red) with the pits of simulated scenarios (in gray).

In the following Figure 6 is shown the amount of ore contained in each of the incremental pits obtained by optimizing the kriging model and selected simulated scenarios.
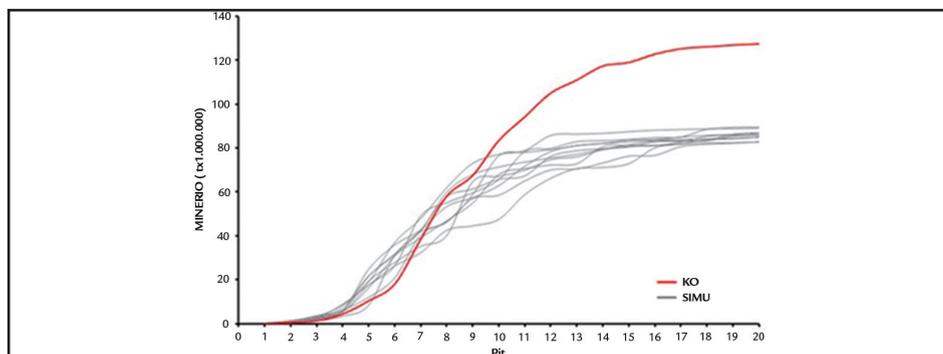

Figure 6
Comparison of ore mass contained in the kriging model pit with pits simulated.

By analyzing the previous figure, it is easy to see that the kriging model pit stands out from the others by having a greater amount of ore and, consequently, a lower strip ratio. Such disparity was provided by the known smoothing effect of ordinary kriging, where low values were overestimated and high values were underestimated.

By performing smoothing, the kriging brought a certain amount of material that should be below the cutoff grade for higher grade values, transforming part of the low grade material to ore. This explains the greater amount of ore presented in the graphic of the previous figure.

The lack of low values reproduction for the estimate was evidenced by

the contained ore graphic, but the lack of high values reproduction can easily be seen in the following figure 7, where the absence of very high grade blocks (responsible for a large increase in the value of the NPV) provides a low NPV for the pit of the model estimated by ordinary kriging compared to the pits of simulated models.

Figure 7
NPV comparison of kriging model pit with pits simulated.
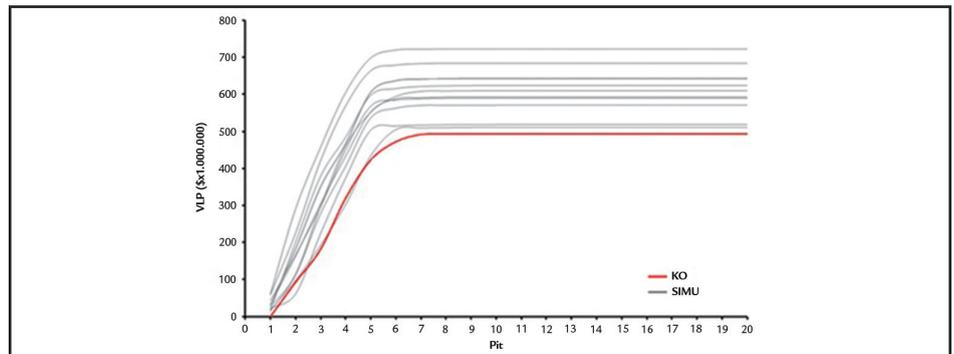
4. Conclusion

Approaching the geological uncertainty with the application of geostatistical simulation techniques proved to be an essential and indispensable tool in the evaluation of long term mine planning projects. Although it is not

widely used, quantifying the geological uncertainty with the use of geostatistical simulation can considerably reduce the possible risks associated with some project factors.

As the shown in the results, when

only considering the model estimated by ordinary kriging as absolute truth, there is a great risk that the estimates of net present value (NPV), quantity of ore and strip ratio of the selected final pit do not reflect the reality.

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