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Investigations on Complex Acoustic Modes of Rocket Engines Combustion Chambers for Damping Allocation

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ABSTRACT

Combustion instability can severely impair the operation of many kinds of combustion engines. Acoustic resonators are widely used to suppress the pressure oscillations caused by the coupling between the combustion process and the combustion chamber acoustic modes. Combustion chambers with subsonic flow in its inlets and outlets, like gas turbine combustors, exhibit some acoustical damping due to the presence of openings. In such chambers, the acoustic modes are complex. In a complex mode, the antinode regions can be shifted from its position in the corresponding real mode. In this work an experimental acoustic modal analysis of a cavity with an opening was performed. Acoustic frequency response functions were obtained by using a volume acceleration source, a microphone and a data acquisition system. The PolyMAX algorithm was used to estimate longitudinal modes in its real and complex versions. A comparison was performed and the results show that, for some modes, the antinode region placement could change reasonably. This suggests that the use of complex modes for location of antinode regions provides more accurate results and consequently could be a better way to identify positions, where resonators provide maximum damping in order to minimize combustion instability in subsonic combustion chambers.

Keywords: Modes; Standing waves; Acoustic measurement; Combustion chambers; Cavity resonators.

INTRODUCTION

Acoustic systems that have elements capable of dissipating the energy of a pressure field have some degree of damping. Damping can be caused in an acoustic cavity by the internal fluid properties or by resistive boundary conditions, like: i) absorbing materials placed on cavity walls; ii) liners or acoustic resonators; iii) resistive radiation of an opening in the cavity walls. These boundary conditions generate acoustic energy dissipation and are related to the resistive (real) part of the complex acoustic impedance of the material. This attenuation is observed in an exponential decay of the pressure and particle velocity, as a function of time and space (Fahy 2001).

Complex natural modes are inherent to acoustic systems with some degree of damping. In this way, all acoustic systems present complex modes. However, the intensity of the damping of a mode influences the ratio between the magnitudes of its real and imaginary parts.

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Combustion chambers with a supersonic nozzle, like the ones of liquid rocket engines, behave acoustically as a closed cavity (Laudien *et al.* 1995) since the nozzle throat region reflects the acoustic waves as a rigid wall and then, in this case, no damping is added due to the nozzle. However, in a combustion chamber where the flow passing through the nozzle is not supersonic, like a gas turbine combustor (Bellucci 2009), the nozzle opening brings damping to the acoustic system. Therefore, when performing a numerical analysis of such acoustic system, one must consider the impedance of all opening regions in the cavity boundary conditions in order to properly estimate the chamber acoustic modes, as pointed out by Lamarque and Poinsot (2008).

The occurrence of combustion acoustic instability motivated the use of devices capable of introducing acoustic damping in combustion chambers. This phenomenon can occur in many combustion systems like stationary and airborne gas turbines, ramjets, rocket engines, among others. It consists of undesirable oscillations in the combustion chamber internal pressure and is caused by the coupling between the combustion process and acoustic waves. Acoustic resonators are a type of device capable of attenuating acoustic pressure fluctuations in a limited frequency range around a specific mode and are widely used in liquid rocket engines and gas turbines combustion chambers (Culick 2002; Frendi *et al.* 2005). In Bellucci (2009), it is shown that the maximum acoustic damping that Helmholtz resonators could add to a gas turbine combustor occurs when the resonators are placed close to the antinode regions of the mode of interest.

In the analysis of combustion chambers with damping, errors in the determination of node and antinode regions could be induced by not considering the modes as complex. One way of estimating acoustic complex modes is by using experimental modal analysis. Acoustic experimental modal analysis has been developed since the 1980s (Nieter and Singh 1982) and the need for developing acoustic sources (Rossetto *et al.* 2000; LMS International 2004) has contributed in this way. A primary study (Guimarães *et al.* 2011) showed that it is possible to perform experimental and numerical acoustic modal analysis by using the same tools as those used in classical structural modal analysis.

It is possible to describe the combustion chamber acoustics by using cold test procedures. Acoustic dynamics in combustion environments are obtained by shifting the cold test resonant frequencies by a scalar factor defined by the ratio of sound velocity at the cold test temperature and at real operation temperature (Laudien *et al.* 1995).

In this work, the presence of complex acoustic modes in acoustic cavities with openings is studied by using analytical methods for simple duct and experimental identification for a cavity with a more complicated shape. The node and antinode regions for the complex and its equivalent real modes are determined and the importance of considering such modes when selecting places for acoustic resonators in combustion chambers with subsonic flow is also addressed.

Complex modes

The homogeneous Helmholtz equation to the one-dimensional problem (plane wave) in the x direction is defined as Eq. 1:

$$\frac{\partial^2 p(x)}{\partial x^2} + k^2 p(x) = 0 \tag{1}$$

where p is the pressure and k is the wave number.

Considering a duct with length L and rigid extremities (closed-closed duct), the particle velocities at the boundaries ($x = 0$ and $x = L$) are zero. It is possible to obtain its natural frequencies f_n by assuming a harmonic solution to the pressure function and applying the boundary conditions to Eq. 2 (Fahy 2001):

$$f_n = \frac{nc}{2L_t} \tag{2}$$

for $n = 1, 2, 3, \ldots$ Each spatial distribution of the pressure (pressure mode) is related to one natural frequency, as shown in Fig. 1 (left) and is given by Eq. 3:

$$p(x) = 2\tilde{A}\, cos(n\pi x/L_t) \tag{3}$$

where $\tilde{A}$ is a complex constant.

Each one of the *n* distributions are called normal (or real) mode (or mode shape) of the cavity. It represents the natural shape of pressure responses to each natural frequency.

Considering the right extremity of the duct being composed by a complex impedance, dissipation (attenuation) of the acoustic energy occurs due to the resistive (real) part of the impedance at the termination. This resistive attenuation leads to an exponential decay of the pressure and the particle velocity, as a function of time. This behaviour can be modelled by a complex frequency $w/c = w + jd_t$ (Fahy 2001), in which d_t represents the imaginary part of the complex frequency.

The pressure spatial distribution to each natural mode can be verified in Fig. 1 (right) by the resultant equation (Eq. 4):

$$p(x) = 2\tilde{A}[cos(kx)\,cosh(\delta_t x/c) - jsen(kx)senh(\delta_t x/c)] \tag{4}$$

Those functions represent complex modes, in which the pressure phase angle varies continuously in relation to the position. This behaviour differs from the real modes, in which the phase angle varies in steps of ± 90° (or 0 and 180°). The phase angle of the mode in position *x* is given by Eq. 5:

$$\theta(x) = arctan[-\,tan(kx)\,tanh(\delta_t x/c)] \tag{5}$$

Complex modes are commonly observed in systems where damping acts locally, i.e., in cases the damping distribution is not uniform. Regarding acoustic systems, an experience (Augustinovicz 2012) showed that, when absorbing material is installed uniformly inside a cavity, real modes are noted. When absorption is located in some regions (and not uniformly), the modes become complex.

The first three acoustic complex modes of a tube are presented in Fig. 1 (right) for the case where the duct right end presents complex impedance with nonzero real part. Comparing those modes with the real modes (left in Fig. 1), one can verify that the variation of the node position for the complex mode doesn't allow the identification of a well-defined zero pressure point or pressure node. One can observe a "throat" with some thickness, indicating that, to each phase angle, the node is located in a different position. In an animation, this behaviour is understood as travelling waves (Augustinovicz 1992), in contrast to a standing wave from the real mode, in which the node doesn't vary its position with the phase variation.

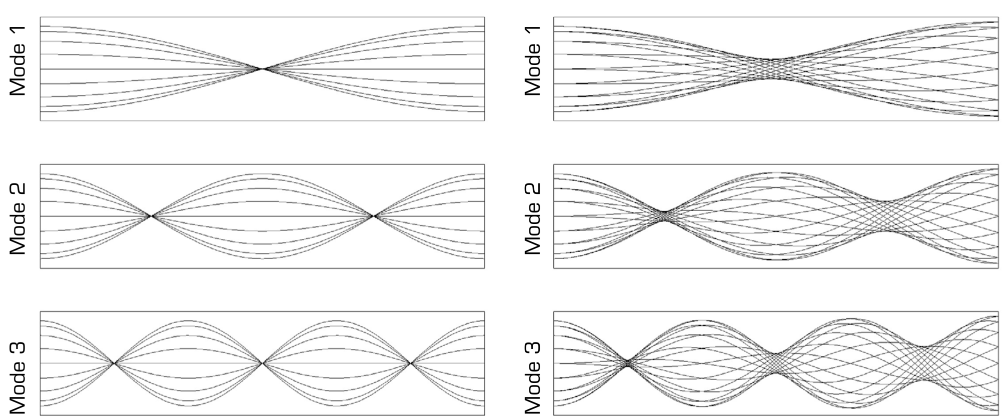

Figure 1. Acoustic modes of a closed-closed duct (left) and closed-absorbed duct (right).

Experimental acoustic modal analysis

Due to the development of commercial acoustic sources, experimental acoustic modal analysis can be a suitable choice in view of extracting the acoustical frequency response function (FRF). In addition, the mathematical approach of the modal parameters extraction of structures can be applied to acoustic systems, considering their linear nature.

There are many methods applied on experimental data curve fitting in order to match a parametric model. Among them, the PolyMAX (LMS International 2006) is considered the evolution of the Least-Squares Complex Frequency-domain (LSCF) estimation method, which uses the complex FRF data into the band of interest. The measured FRFs matrix [H(ω)] is represented the following model in Eq. 6:

$$[H(\omega)] = \sum_{r=0}^{N} z^r[\beta_r] \ \left(\sum_{r=0}^{N} z^r[\alpha_r]\right)^{-1} \tag{6}$$

where $[b_r]_{l,m}$ is the matrix of polynomial coefficients for the numerator, $[a_r]_{m,m}$ is the matrix of polynomial coefficients for the denominator, l is the number of responses and m is the number of excitations. N is the model order, which is developed in the discrete domain $z = e^{jwDr}$.

The poles l_i and the modal participation factors $\langle h_i \rangle$ are calculated from the coefficients $[a_r]$. The mode shapes $\{y_i\}$, the lower residuals LR and upper residuals UR are obtained from the following partial fraction model (Ewins 2000), to each mode n (Eq. 7):

$$[H(\omega)] = \sum_{i=0}^{n} \left(\frac{\{\psi_i\}\langle\eta_i^T\rangle}{j\omega - \lambda_i} + \frac{\{\psi_i^*\}\langle\eta_i^H\rangle}{j\omega - \lambda_i^*} \right) - \frac{LR}{\omega^2} + UR \tag{7}$$

The indexes *, ^{T} e ^{H} indicate the conjugate complex, transposed, conjugate complex transposed from the referred matrix, respectively. The residuals consider, in the model, the influence of the modes outside the delimited frequency band. The poles are identified in conjugate complex pairs and are related to the natural frequencies W_i and the damping factor Z_i from each mode i, as Eq. 8:

$$\lambda_i = -\zeta_i\omega_i + j\sqrt{1-\zeta_i^2}\,\omega_i \tag{8}$$

METHODOLOGY

The liquid propellant rocket engine mock-up, showed in Fig. 2, was used in previous studies of acoustic behaviour of liquid rocket engine combustion chamber (Pirk *et al.* 2010). In the left extremity of the original engine, the face plate and injectors are located. In this mock-up, the face plate was simulated by a plane surface, where an acoustic source was installed. Since the gas stream velocity passing across the nozzle is supersonic on operational conditions, the chamber behaves acoustically as a closed cavity. Therefore, in acoustic tests at room temperature, a cover needs to be included on the throat in order to reproduce properly the cavity actual boundary conditions. In this work, this mock-up was analysed experimentally without a cover on the nozzle throat aiming to introduce damping, due to its resistive radiation, and cause the presence of complex modes.

The experimental acoustic modal analysis was performed. This procedure starts from the acquisition of excitation signal from the acoustic source and response signal from the microphones. After that, the frequency response functions (FRF) are calculated and then the dynamic parameters (natural frequencies and mode shapes) are obtained by using the presented theory. The experimental setup was performed to identify only the longitudinal modes.

The PolyMAX algorithm considers two types of mode identification: real modes and complex modes estimations. Both estimators were calculated, presented and compared.

Experimental setup

The mid-high frequency volume acceleration source (LMS International 2004) produces a voltage signal proportional to the volume acceleration [m^3/s^2] variation, with a nominal frequency range from 200 up to 8,000 Hz. The source nozzle was installed in the mock-up plane surface (left side), as shown in Fig. 2.

The cavity excitation was provided by a white noise function generator, a power amplifier and the referred source. The microphone was supported by a thin rod to be positioned in each measurement point inside the cavity. The pressure oscillation inside the cavity was captured and registered into the data acquisition system SCADAS using the Test LAB software (LMS International 2006). The sampling rate of 8,192 samples/s (frequency range of 3,200 Hz) and a length of 6,400 points assured a frequency resolution of 0.5 Hz.

The acoustic cavity was discretized in seven points of microphone measurements along the central longitudinal axis (Fig. 2). The point "0" represents the driving point FRF, since it is the position of an ideal omnidirectional point source, called acoustic centre. The other points (1 to 6) are placed 50 mm from each other, from the face plate in the same axis line.

The measurements were done by using the free run method and stopped automatically after 100 averages, for each one of the 7 FRF. At each new measurement point, the auto-range process was applied to equalize the input amplifier gain and, then, to optimize the signal-to-noise ratio.

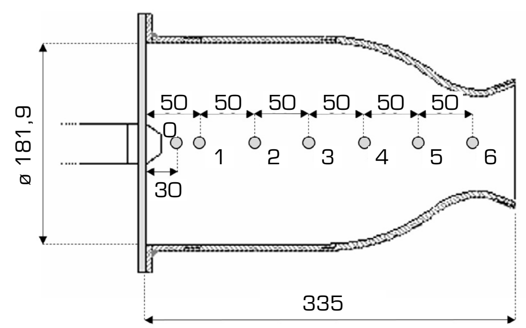

Figure 2. Chamber dimensions, source and microphone positions – dimensions in mm.

RESULTS AND DISCUSSION

The seven measured frequency response functions are presented in Fig. 3. The presence of three resonance peaks can be easily noted. In some functions, there is some evidence that a fourth resonance exists around 1700 Hz. Such spectral function set was considered as the basis to perform a modal parameter extraction procedure.

The measured FRF for the microphone location 6 is presented in Fig. 4. Well defined peaks are noted in module plot. In phase angle plot, such peaks coincide with the phase inversions in the resonances and antiresonance frequencies values. The respective coherence function presents high levels (near the unity) of over 100 Hz. Below this frequency the coherence is low due to the low level of source energy.

By using the PolyMAX algorithm, those functions were processed at Test LAB software to extract the modal parameters. It was possible to achieve the real and complex modes by varying the type of modal shapes (residues) calculation.

Since the longitudinal modes are the only modes considered, the microphone positions were distributed in a line across the symmetry axes of the cavity. The first four complex modes of the motor cavity and its respective natural frequencies can be noted in Fig. 5.

The mode shapes representation considers the module being plotted with increment of 15° in phase angle to each curve up to it achieve 360°.

The effects presented in theory regarding complex modes, illustrated in Fig. 1, could be verified: as the phase angle varies along the duct length, the node is located in a different position to each phase angle. Two main effects observed in Fig. 5 can confirm this behaviour: (i) in mode shapes 3 and 4, there is no node point clearly observed but only a "throat" where an ideal node is supposed to exist; and (ii) the mode shape lines cross each other in certain region, in opposition of the "parallel" lines of real modes.

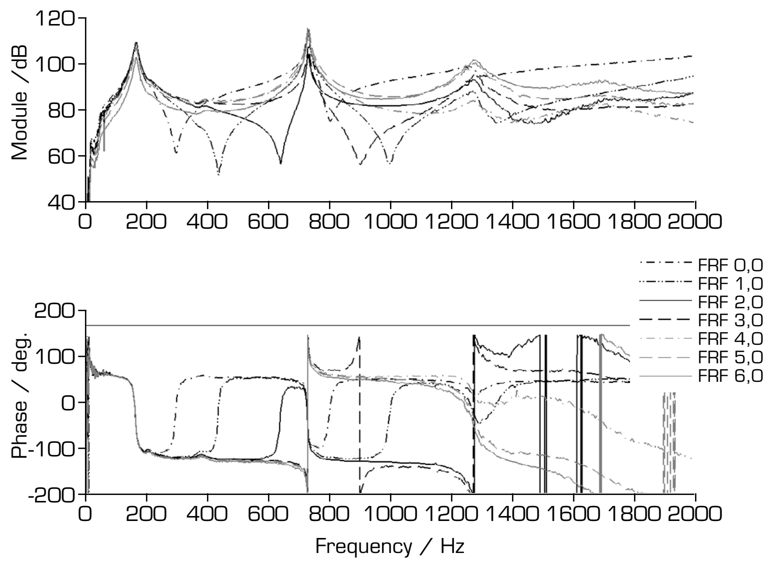

Figure 3. Frequency response function set used to the modal analysis.

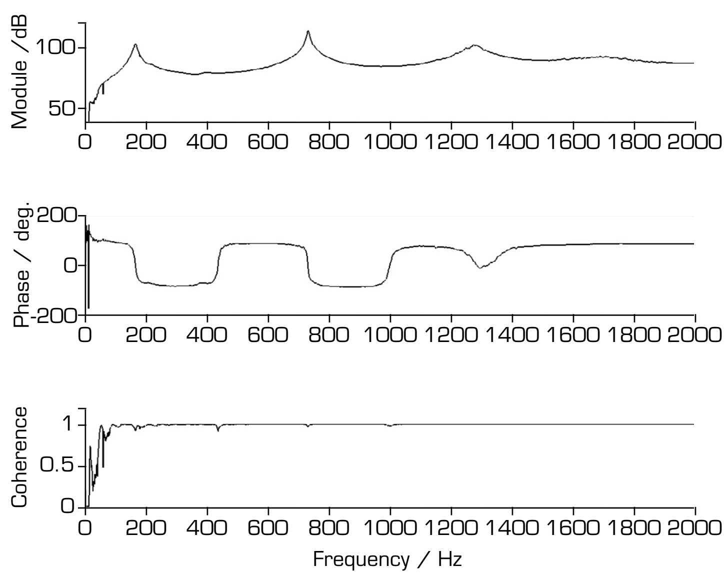

Figure 4. Typical FRF: module, phase angle and coherence functions from $FRF_{6,0}$.

The comparison between the real modes estimation and complex modes estimation performed by means of the FRFs curve fitting could be verified in Fig. 6 (third mode) and Fig. 7 (fourth mode). In the third mode, complex and real modes are similar. The "throat" regions in the complex mode are related to the node regions on the real mode shape.

However, in the fourth mode comparison (Fig. 7), the degree of similarity between the real and complex mode shapes is very low. While the maximum pressure amplitude is observed close to de face plate (left) in the complex mode shape, for example, the pressure amplitude presents very low levels in the same region for the real mode shape case. Considering a second region for analysis between 0.15 and 0.20 m of the chamber length, high pressure location in complex mode is related to low pressure values in the real mode estimation and vice versa.

Some investigation has been performed about the low number of degrees of freedom employed, which was equal to seven. The higher order mode shapes (as the mode 4) present low spatial definition, as expected. Nevertheless, this feature is present in both estimation process (real and complex) and the comparative analysis remains suitable.

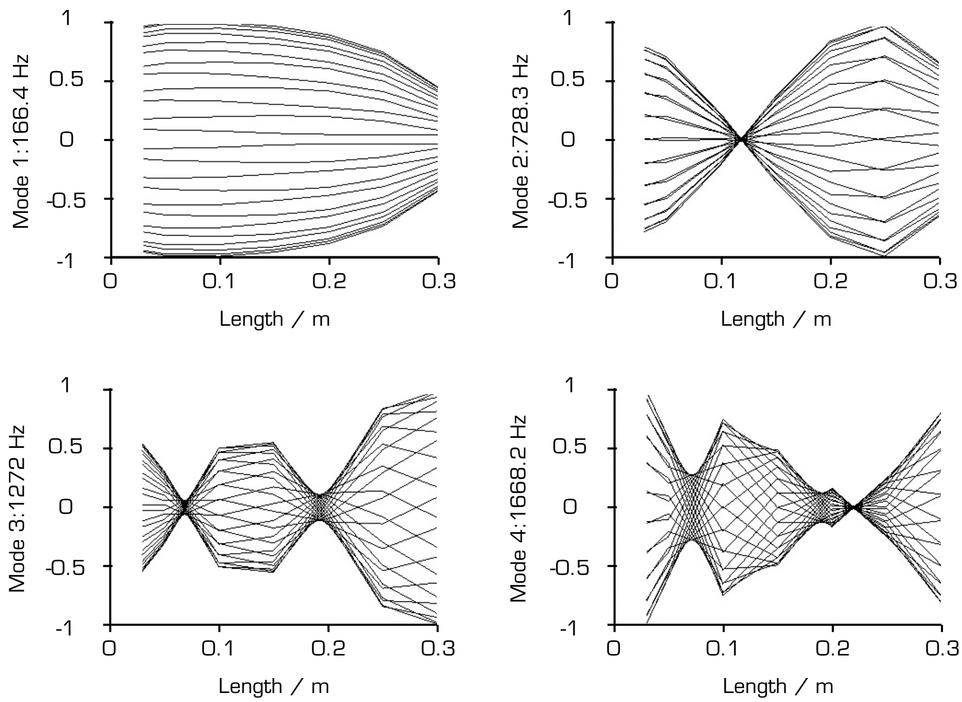

Figure 5. Complex modes and natural frequencies.

The huge difference between the real and complex estimations for the fourth mode is indeed related to very high effect of local damping and that is physically explained by the radiation resistance of the nozzle aperture (right end). Also, the damping increases as the mode order increases. It can be observed in the resonance peaks in the FRFs (Fig. 3) for the first 4 natural frequencies: the peaks became smoother as the order increases.

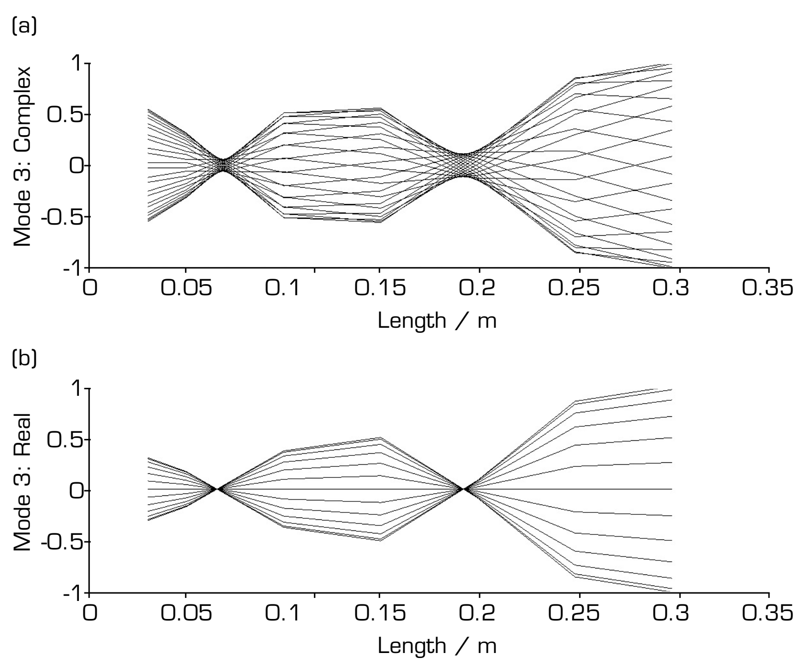

Figure 6. Complex (a) and real (b) third mode.

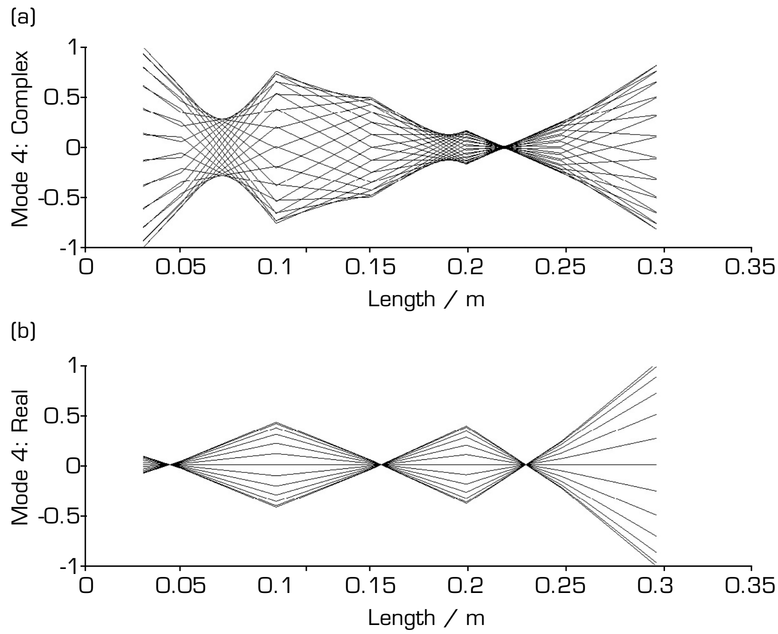

Figure 7. Complex (a) and real (b) fourth mode.

CONCLUSIONS

Complex acoustic modes can easily be noted in the literature where simple shape cavities, such as ducts and pipes, present one absorbing end. In this work, the graphical representation for acoustic mode shapes was implemented, comparing a duct with closed ends (and its respective real modes) and a duct with one absorbing end (and its respective complex modes). It could be possible, in this way, to recognize the complex modes effects obtained by analytical methods.

Those effects could be observed in the complex modes, estimated experimentally in this work, for an acoustic cavity with no such simplified shape. In addition, there was no absorbing material in the cavity, but a radiation resistance due to the nozzle shape and chamber termination, that comprises the damping (energy dissipation) present in the model.

Considering the PolyMAX algorithm, it could be possible to estimate the mode shapes using complex or real procedures. Real (or normal) modes are normally used to compare/validate conservative numerical models, i. e., models for which dissipation is not considered. However, as the damping became significant in the behaviour observed (mainly if experimental results are considered), the real mode estimation does not represent properly the mode shapes, as can be noticed in the results.

In fact, real mode estimation represents a simplified form to approximate complex modes to real modes and it is allowed in commercial algorithms, as PolyMAX. Such approximation must be used with caution, as it could lead to a mistake in the analysis if the complexity of the modes is pronounced (as the fourth mode in this study). In the cavity studied, estimating the fourth mode (in Fig. 7) as real or complex gives very different mode shapes and antinode regions.

Although the cavity studied does not represent a subsonic combustion chamber geometry, it could be observed that the presence of openings in a cavity of a subsonic combustion chamber adds an amount of damping that makes the chamber acoustic modes complex. The difference between the antinode regions placement and shapes of complex modes and real modes varied significantly from mode to mode. In this way the results indicate that the presence of complex modes could be an important issue when designing a system of acoustic resonators to suppress acoustic instabilities in subsonic combustion chambers.

AUTHORS' CONTRIBUTION

Conceptualization: Guimarães GP and Pirk R; **Methodology:** Guimarães GP and Pirk R; **Investigation:** Guimarães GP, Góes LCS, Pirk R and Souto CA; **Writing – Original Draft:** Guimarães GP, Pirk R and Souto CA; **Writing – Review and Editing:** Guimarães GP and Souto CA; **Funding Acquisition:** Góes LCS and Pirk R; **Resources:** Góes LCS and Pirk R; **Supervision:** Góes LCS and Pirk R.

DATA AVAILABILITY STATEMENT

The data will be available upon request.

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Not applicable.

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