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Genetic algorithms applied to an evolutionary model of industrial dynamics☆

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Abstract

In order to verify the effects of machine learning in a market structure, an evolutionary model containing firms that use a genetic algorithm to decide their investment in innovative R&D was developed. These firms share the market, with two other types of firms, those with a fixed rate of investment and those with random strategies. A model of industrial dynamics was implemented and simulated using several population distributions of the three types of firms. The availability of external credit and the length of learning periods were evaluated and their effects, in the market structure, analysed. The simulations results brought contrasting findings when compared to previous works, as it confirmed that machine learning led to market dominance, but the same did not occur when considering the improvement of technological efficiency and social welfare.

JEL classification: L16; C63; C61

Keywords: Genetic algorithms; Agent-based modeling; Evolutionary model; Industrial dynamics

1. Introduction

The most known model of industrial dynamics is the one developed by Nelson and Winter (1982). In this model, each firm makes an R&D investment in order to improve their productivity, which is divided between the search for innovation and imitation. There are two different types of firms, the ones that only imitate and those which innovative and imitative processes occur simultaneously. The level of productivity achieved in one period is carried to the next, beginning the industry's evolutionary process.

According to the model created by Nelson and Winter (1982) the innovation is defined as the search for new means of production and processes improvement. The imitation consists of endeavouring to copy the market's best practice (highest level of productivity). As the investment in R&D is a fraction of the capital stock, the crucial factor for innovation and imitation is the firm's size, because a larger value of capital stock means a higher probability of success.

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The two-way relationship between market structure and innovation must be highlighted. Also, the choice for innovative investment is generally less profitable than the choice for imitative investment, this is due to the initial structure, which inflicts higher research costs to the innovative firms.

When considering a situation of incomplete information, each firm does not know the others' decisions, therefore, it's not possible to forecast what is the best action. Schaffer (1989) discusses the idea of non-profit maximizing firms and applies concepts of the "spite effect" phenomena. Which consists of the fact that a subject can choose a set of strategies that leads to a minor gain if the other individual suffers a more relevant loss. Arifovic (1994) argues that when a time progression exists, the agents' knowledge, acquired on the previous levels, must be considered. Therefore, this is the context of learning: using previous experiences to guide the next decisions. Vriend (2000) classifies learning using two basic formats, individual and social. The first, consists of the set of information independently obtained, for example, by a failure/success method. On the other hand, the latter consists in the transmission of behaviours and characteristics from one individual to the other, this could occur, for example, by imitation. According to Vriend (2000) both learning approaches are relevant and must be analysed together, because different results may be found when those are computed in isolation. Those differences are due to the "spite effect".

One way to implement machine learning, first developed by Holland (1975), is by using genetic algorithms. They are evolutionary algorithms based on the concepts of natural selection, which means that the environment selects the fittest beings and their offspring inherits genes that allow the continuity of species evolution. Genetic algorithms enable the search, from an initial set of strategies, for approximated solutions to the optimization problems. The initial strategies are tested, then combined and/or modified, in order to search for new and better strategies, creating a new generation, resulting in an iterative and, consequently, evolutionary process.

Shubik and Vriend (1999) use dynamic programming to relate game theory approaches and behavioural simulations. Even when agents have a considerable amount of information, if they are initially far away from the rational expectations equilibrium, they might not reach the equilibrium point. On the other hand, if the agents have little information in order to forecast the future, they could make the assumption that some characteristics from the past will remain constant. By doing so, they use their past experiences to base their future actions, this allows a classification system to learn and recognize good action patterns, finding implicit solutions to dynamic programming problems. Therefore, even when an explicit solution cannot be found, an adaptive algorithm can make the calculation and find, through simulation, a solution.

Simulations are very useful when applied to evolutionary economics. Kwaśnicki (1999) analysed different simulation approaches for the economic development, comparing both Schumpeterian based models, for example the model of Nelson and Winter (1982), and agent-based approaches. An agent-based model does not have all features completely specified, thus there is one defined set of strategies, that allows the agents to make choices in the search for survival. When economically applied, both models act similarly, agents start their interactions without any previous knowledge and, step by step, build their database through learning. According to Kwaśnicki (1999), Schumpeterian models have the advantage of making the ability to relate real time and simulation time possible, while in agent-based modelling, the understanding of this relationship (changing dynamics and time) is more difficult. Therefore, the author suggests considering both modelling strategies together.

In order to compare different types of R&D investments, one using a fixed rate and the other defined by a genetic algorithm, Yildizoglu (2002), uses the two strategies in a simplified Nelson and Winter (1982) evolutionary model and verifies their effects in social and technological welfare, evaluating the relevance of using genetic algorithms. In each time step, firms that use a fixed R&D rule, regardless the market situation, do not change their method of investment. On the contrary, firms, that use genetic algorithms as a learning tool, have variable R&D investment dependent on the industry's conditions.

Therefore, in a genetic algorithm, an agent has a set of strategies, known as chromosomes, which have the same number of genes, identical in length. During the agent's lifetime, those genes are constantly modified (evolution) as consequence of the gained experience and results. Yildizoglu (2002) has employed the genetic algorithm, at the end of each generation, to define the percentage of profit invested in innovative R&D. A generation is the necessary period to test all strategies. As the new chromosomes depend on the actual industrial dynamics, the behaviour of each agent will depend on the actions of its competitors. Thus, as the new strategies are not necessarily short-term investments, they must be tested along some time steps. For each strategy, the average gross profit rate was computed, for the purpose of evaluating its fitness.

Furthermore, the convergence of the investment rate, Yildizoglu (2002), was found in firms that used the genetic algorithm and the presence of learning resulted in higher technological and social efficiency, in fact, the R&D activity is dominated by the firms that have used the genetic algorithm.

This paper aims to implement an evolutionary industry model, as proposed by Nelson and Winter (1982), adding some of Yildizoglu (2002) premises and doing some new assumptions and changes in parameters to verify the efficiency and effectiveness of using genetic algorithms not only in markets with fixed R&D investment firms, but also, in other markets where firms decide their investment randomly. Among these changes it must be highlighted, that in the present model firms do not exit the market, different learning periods for the genetic algorithm are tested, a new fitness function is used, the external credit influence is analysed, and the investment in R&D is defined as a percentage of the capital stock. It's expected to verify and validate the learning through genetic algorithms and to compare the effects of those parameters modifications on the results.

In order to run the simulation, the model is described using the "ODD Protocol" (Overview, Design concept and Details), then, it is implemented and simulated using an agent-based modelling approach in NetLogo.

The remaining of this paper is organized as follows. Section 2 presents the theoretical framework of the model with a small explanation about the main operators of a genetic algorithm. Section 3 consists of the model description. Section 4 shows simulation results and their discussion. Finally, Section 5 presents the main conclusions and a brief discussion about the findings.

2. Theoretical framework

The economic theory tells that firms' profits in perfectly competitive markets tend to zero in the long run, however, in an oligopoly, there is non-null profits. Consequently, in this case, the entry of new competitors is interesting, but it is necessary for the entrants to surpass entry barriers imposed by the market. Winter (1984) has studied this emergence of new firms by analysing different situations of entry, considering the market conditions that favour the entrance and evaluating the potential of new entrants. The author also studied the market's exit conditions. When a firm goes out of the market, there is an offer reduction resulting in an increase of the product price in the next period.

In a situation of incomplete information, each firm does not know the others' decisions, therefore, it's not possible to forecast what is the best action. Witt (1986) analysed the behaviour of firms in scenarios with lack of information, through the simulation of three different models, each using a specific set of rules. The objective was to verify whether the choices of act as a profit maximiser are dominant when comparing to learning and behaviour adjustment strategies. Thus, Witt (1986) affirms that profit maximization is neither necessary nor sufficient condition to increase the probability of survival for a firm in dynamic markets.

The genetic algorithms can be applied, for example, to a firm's optimization problem that has limited or absent information. Thus, the most efficient type of investment is unknown and using the algorithm allows seeking strategies in order to get better results. The genetic algorithm simulates the situation where firms have few or zero information about the market and, from an initial set of actions, the results are observed, and new strategies created. So, it is important to understand two main features: the industrial dynamics and learning. The former consists of how the entrance and exit of firms occur, how much each firm invests in R&D and how the investment is made (innovation or imitation). The latter refers to how the databases of knowledge, which guide the firms' choices, are built.

Arifovic (1994) analyses different learning methods, using a cobweb model to compare the performance of genetic algorithms and other methods. The algorithms have three different genetic operators: reproduction, crossover and mutation. Reproduction consists of making copies of the most fitting strategies (chromosomes), because those have a higher probability of generating a new chromosome that would be useful to the genetic evolution. The crossover consists of getting part of the genetic code of two different strategies to create a new chromosome. Next, the mutation operator takes over and there is a random gene mutation, therefore, some elements of the strategies could be modified, generating the final new list of strategies. In some cases, a fourth operator might be used: election, which consists of testing the newly generated chromosomes before allowing them to be part of the population. This optional operator allows the elimination of some mutation's negative effects.

According to Beckenbach (1999), several studies consider the genetic algorithm as a function optimizer and in most of the cases, they are used to find the maximum value of a unimodal time-invariant function, which results in the necessity of having an exclusive system of performance improvement for the genetic algorithm. Therefore, this goes against the idea of genetic algorithms being an adaptive search procedure, that tries to solve high complexity decision-making problems, which complexity is derived from environmental uncertainties. Thus, it is accepted that the circumstances must be described by a multimodal time-variant function, without observer interference, to correctly represent the population's survival skills evolution.

Shubik and Vriend (1999) use dynamic programming to relate game theory approaches and behavioural simulations. Even when agents have a considerable amount of information, if they are initially far away from the rational expectations equilibrium, they might not reach the equilibrium point. On the other hand, if the agents have little information in order to forecast the future, they could make the assumption that some characteristics from the past will remain constant. By doing so, they use their past experiences to base their future actions, this allows a classification system to learn and recognize good action patterns, finding implicit solutions to dynamic programming problems. Therefore, even when an explicit solution cannot be found, an adaptive algorithm can make the calculation and find, through simulation, a solution.

As a kind of evolutionary algorithm, the genetic algorithms allow learning through experience. Basically, new strategies are generated from the previous ones to seek and find new solutions with a better fit. Thus, it consists of an optimization of the fitness function through the algorithm's implementation. According to Mitchell (1998), a genetic algorithm is more efficient and competitive than other classic methods, if the search space is large, neither smooth nor unimodal, not well known, the fitness function has too much noise or if the procedure does not demand a global maximum to be found. But, either way, the key success factors of a genetic algorithm are the encoding of possible solutions and operators, the settings features and the definition of the success criteria.

The most traditional way of encoding system, according to Mitchell (1998), is the binary encoding, which consists of converting each option from the set of possible choices into a "string" (chromosome) containing 0 or 1 values, each one of these chromosome's elements is a gene. Then, after encoding, some solutions are chosen and tested, the procedure is followed by the genetic operators (selection, crossover and mutation), in order to develop the new generation of chromosomes.

2.1. Selection

According to Mitchell (1998), the selection operator is the next step after the encoding decision. It consists of selecting chromosomes that are going to have their genetic material transmitted to the next generation. There are several ways of selecting those candidates, but for the scope of this paper, only the elitism and roulette wheel methods are going to be undertaken:

2.1.1. Elitism

Elitism consists of maintaining a certain number of chromosomes in the next generation, guaranteeing that those individuals' characteristics are not going to be lost after crossover and mutation procedures. In the case of wanting to maintain only the individual with the best fitness, this member is selected as part of the new chromosomes generation. This way, not only the strategy that obtained the best result will be able to is not going to be lost and will still.

2.1.2. Roulette wheel method

Also known as fitness proportionate selection, the roulette wheel method, consists of setting selection weights for the chromosomes proportionate to their fitness. It is the same as giving to each individual, a roulette wheel slice that has its size proportional to their fitness. Next, after each spin, one individual is selected to be part of the parents' population. According to Mitchell (1998), one way to implement this method is:

Step 1 – Obtain the value T, which is the sum of all chromosomes' fitness values.
Step 2 – Draw a random number, r, between 0 and T.
Step 3 – Orderly sum each value of fitness, accrediting each to its respective interval range.
Step 4 – Select as part of the parents' population the individual that is responsible for the interval range that contains the value r.
Step 5 – Repeat steps 3 and 4 until all the necessary parents are selected.

Table 1
Single point crossover.

Parents	Crossover point	New generation
1 0 1 0 0 1 1	1 0 1 0|0 1 1	1 0 1 0 1 0 0
0 1 1 0 1 0 0	0 1 1 0|1 0 0	0 1 1 0 0 1 1

Table 2
Example of chromosomes mutation.

Chromosomes	Before mutation	After mutation
1 0 1 0 0 1 1	1 0 **1** 0 0 1 1	1 0 **0** 0 0 1 1
0 1 1 0 1 0 0	0 **1** 1 0 1 **0** 0	0 **0** 1 0 1 **1** 0

After selecting both parent chromosomes the crossover and mutation operators can act to define whether and how they are going their offspring, the new strategies.

2.2. Crossover

In biology, theories related to the species crossover justify the transmission and emergence of characteristics through the reproduction of two individuals. There is always exchange of genetic material when two beings sexually reproduce, and their offspring has genes from both parents, creating a completely new set of characteristics.

The crossover operator of genetic algorithms acts similarly, making evolution possible, due to the mixing of two chromosomes' genes. According to Mitchell (1998), the crossover consists of using parts of the genetic material of one chromosome and adding some genes of the other. An example of a single point crossover can be observed in Table 1, part of each 7-gene individual is selected and added to the other, forming two new individuals.

2.3. Mutation

Mutation is, in biological terms, alterations of individuals' genotypes, in other words, they have genes that did not originate in their ancestors. Those new characteristics are obtained from a failure in the genes reproduction and create new features that can be passed to the individual's future offspring, modifying, in the long-term, the species attributes.

Therefore, the genetic operator of mutation is essential to the variability of individuals and, as stated by Mitchell (1998), it avoids the premature convergence of the individuals to a non-optimized solution. On the other hand, if the mutation operator is too present, it could lead to an intense variability of the sample, making the convergence to the best solution impossible.

In practice, this operator consists of changing the chromosome's genes. In a binary encoding, the mutation stage would be the probability of one element of the string being modified. Thus, if a gene that should be an element 0 suffers mutation it would be modified to 1 and vice versa. As shown in Table 2, mutation can occur as a draw, which the success changes one chromosome's random gene, or as a draw for each gene, which allows more than one to suffer mutation in the same round.

3. Model description

The model consists of applying a genetic algorithm as an intelligence source for an evolutionary model based on Nelson and Winter (1982). As it was proposed by Yildizoglu (2002), some firms use a genetic algorithm to define their investment in innovative R&D, however the rates instead are calculated on the capital and not on profits like Yildizoglu (2002), afterwards their performance is compared to those of two other types of firms. The ones with fixed innovative strategies and the ones that have zero intelligence, defining their innovative investment randomly at the end of each period.

In addition, this model also differs from the one developed by Yildizoglu (2002) as firms do not exit the market, various learning periods for the genetic algorithm are tested, the fitness function is defined as the average net profit rate, the external credit influence is analysed, and the investment in R&D is defined as a percentage of the capital stock, while Yildizoglu (2002) uses a fraction of the previous period gross profit. The design concepts of the model, described in Section 3.1.4, explain the reasons behind each of these implementation decisions.

In order to facilitate the comprehension and reproduction of the model, it is described by an ODD(Overview, Design concept and Details) protocol, as suggested by Railsback and Grimm (2012), in order to detail the agent-based model used. The simulations are made using NetLogo software developed by Wilensky (1999).

3.1. Model's ODD protocol

3.1.1. Purpose

Verify the effects of intelligence on the market structure through the introduction of genetic algorithms in an evolutionary model. In order to do so, first, firms that use a genetic algorithm to define their innovative investment share the market with firms that use fixed strategies. Then, in another scenario genetic firms share the market with firms that do not use any kind of intelligence, just defining their investment randomly. Finally, the performance of each of these strategies is evaluated, by verifying the relevance of using a genetic algorithm in an evolutionary model.

3.1.2. Entities, state variables, and scales

There are three types of agents (entities): firms that use the genetic algorithm (*genFirms*); the *nwFirms*, that use the Nelson and Winter (1982) investment rate; and those with random strategies of investment(*rndFirms*).

All the firms have fixed imitative R&D investment, r_{im}, as proposed by Nelson and Winter (1982), but the *genFirms* have the following features: variable innovative R&D investment, therefore, its r_{in} is altered according to the genetic algorithm. Then, the particular characteristics of *nwFirms* are: R&D investments in innovation and imitation being fixed, thus, they follow the Nelson and Winter (1982) values r_{in} and r_{im}, respectively. Finally, the *rndFirms* are different because, even though they also have a variable innovative R&D investment, it depends on a stochastic process – randomly generating a r_{in} value, which is limited to a maximum of the *nwFirms* innovative R&D value times a multiplier.

Every firm has as variables: capital stock (K) and productivity (A). As it was previously mentioned, *genFirms* and *rndFirms* also have as variable the percentage of capital invested in innovative R&D, r_{in}.

On their model, Nelson and Winter (1982), established that the simulation duration is 100 time steps with each period representing a trimester. So, it is necessary to adjust the time scale, for the purpose of allowing the genetic algorithms iterative/evolutionary process to occur. Yildizoglu (2002) fixates those time steps at 6000, making it necessary to divide each trimester of the original model by 60 in order to maintain a similar number of periods. For each situation, 20 simulation runs were executed.

3.1.3. Process overview and scheduling

After the initialisation of the procedures, the agents invest a percentage of their capital stock in R&D, using predefined strategies, they also invest a fixed percentage of their capital stock in imitative R&D, both investments are made to improve the firms' productivity, increasing their profits in the following periods.

The innovation investment rate of the *genFirms* is defined by using an initial set of strategies. The fitness is defined as the average profit rate of the periods in which the strategy was used. These firms update their chromosomes at the generation's end, after all strategies have been tested, therefore, the percentage of the capital invested in R&D changes over time. The whole process of strategy testing and the explanation of how the genetic algorithm works is explained in Section 3.1.7.

The *rndFirms* decide their innovative investment randomly, then, at each round, percentages of capital are drawn and invested in innovative R&D, the maximum value of the draw is defined as five times the percentage of capital that the *nwFirms* invest.

At the end of each generation, firms that use the genetic algorithm can change their strategies, the *rndFirms* change them after each period and *nwFirms* utilise the same strategies during the entire simulation.

3.1.4. Design concepts

Basic principles: a predefined number of firms (32) is generated; the firms use a percentage of their capital stock in imitative and innovative R&D, r_{im} and r_{in}, respectively. In each time step, the firms try to innovate, generate a new productivity level, or imitate, copy the best practice of the market. After each period the firm selects the highest productivity among those obtained by research investment and the previous one. Then, the firm new productivity level is set as that last period highest value.

In the first market, a fraction of the firms has fixed investment strategies, *nwFirms*, the remaining firms are *genFirms* and decide their innovative R&D investment by using a genetic algorithm. The maximum value, this investment can achieve, is equal to five times the fixed percentage of the capital of the *nwFirms*. This limit is necessary to avoid the excessive exposure of the capital stock, which could generate disturbances to the market structure. Led by the search for greater profit rates, the genetic is implemented to select the strategies, percentage of capital invested in R&D, that results in a greater profit. In the second market, the *nwFirms* are replaced by *rndFirms* that define their investment in innovation by means of a stochastic event. The same maximum investment limit of the *genFirms* is set for the *rndFirms*. Each market is simulated using 0%, 25%, 50%, 75% and 100% of *genFirms*.

The profit in one period determines the capital investment of the next. In case of positive profits, the firm has on top of its own profit, a source of external credit (*BANK*), which is predefined as a multiple of the firm profit (0, 1 or 2.5). The next period investment in capital stock ($K_{i(t+1)}$), is, by definition, always non-negative. Thus, even when the profit is negative, the capital stock of the firm will only be decreased by the depreciation rate (σ), as the capital investment would be null.

Yildizoglu (2002) determined that each strategy is to be tested during 5 time steps. Nonetheless, simulations were performed also using other durations of the learning periods for the purpose of verifying the relationship between learning period and market structure.

Applying the concepts of firms' entrance and exit proposed by Winter (1984), Yildizoglu (2002) has assumed that when a firm has constant negative profits, their capital stock diminishes due to the depreciation, if it goes below a minimum value, this firm leaves the market, because it has lost its investment capacity. In the actual model, this parameter is not considered, because maintaining in the market firms with a very low capital does not substantially influence the market structure, as the condition to trigger the firm's exit refers to an amount of capital stock that do not inflict a significant temporary offer reduction, therefore the increase in prices is not relevant enough to be sensed.

Another particularity is the use of Nelson and Winter (1982) original values for the percentage of imitative and innovative R&D using the capital stock as reference. Yildizoglu (2002), on the other hand, uses the profit as reference, estimating the R&D investment percentage. This change is relevant, because it does not only allows the final results to be comparable with the Nelson and Winter (1982) model, but also allows a firm that eventually gets negative profits to not completely losing its capacity to invest in R&D, then they can, for instance, overcome the losses from one period through the improvement of productivity. Using the gross profit, in practical, means that once a firm obtained a negative profit it instantly losses all its capability of R&D and will have its capital stock depleted until it triggers the market's exit condition.

The model developed by Yildizoglu (2002) also sets a minimum rate of investment, feature not present in this current model, because setting this minimum rate constraints the model, as it does not allow the firms that use a genetic algorithm to eventually behave as purely imitators, like some of the firms of the Nelson and Winter (1982) model.

Adaptation: The firms that use genetic algorithm store information about the fitness of each chromosome (strategy) used in that generation. Therefore, they have a certain level of intelligence and their decisions are guided by learning. The fitness of the investment strategy is measured by the average profit rate of the firm. The investment strategies that result in higher profit rates are "winners" and more likely to become starting points for the new strategies that are going to be created by the crossover and mutation mechanisms.

Objective: Using a genetic algorithm, optimize profit using the best trade-off between capital stock expansion and innovation.

Learning: genFirms make use of the knowledge acquired from the results observed after the use of certain strategies. Chromosomes with better results have a greater chance of being chosen in the next periods (selection by roulette wheel) and, therefore, have a higher probability of having their genetic material transmitted to the next generation. The best strategy is kept unchanged and it will be part of the next generation, preventing the loss of the winning strategies (elitism).

Prediction: The companies that use the genetic algorithm assume that the past winning strategies, those that have achieved better results, are more promising candidates to be more successful in the next period.

Sensing: The agents have access to the evolution of their profit data; thus they can infer whether their strategies have succeeded. The agent's "memory" is equal to the number of strategies used in each generation (*Cr*). Each strategy, used over a determined set of periods, has its average profit rate computed and stored. The set of periods is the learning period parameter.

Interaction: Agents do not interact directly, but when they are successful in imitation, they copy the best productivity among market players, that is, they get the highest productivity rate from that period.

Stochasticity: The investment in R&D is divided into innovation and imitation. The result, of the investment and the efficiency of innovation are random events. The innovation consists of a process, in which the innovative success for each investment in R&D is given by:

$$P[d_{in} = 1] = a_{in} \times r_{in} \times K_j \tag{1}$$

where, for the *nwFirms*, $r_{in} = r_{in_{NW}}$, for the *genFirms*, $r_{in} = r_{in_{NW}} \times rd_{ij} \times f$, where f is the multiplication factor of $r_{in_{NW}}$ that limits the maximum value of r_{in} and rd_{i_j} is the percentage of the search space concerning the current chromosome. In the case of *rndFirms*, $r_{in} = r_{in_{NW}} \times RN \times f$, therefore, the probability of innovation success is proportional to the multiplier factor and the *RN* is a randomly generated number within the range [0, 1]. The calibration parameter (a_{in}) is set to 0.125, as proposed by Nelson and Winter (1982).

If this first event is successful, a new productivity value is obtained by generating a random number in the log-normal distribution, with a time-dependent average:

$$\log(\tilde{A}jt) \longrightarrow N(A_0 + (1+\alpha)^{t/f_t}, \sigma) \tag{2}$$

where A_0 is the initial productivity, constant and equal to 0.16, α is set to 0.01 and σ is 0.05; it should be highlighted that the exponential relationship with time, divided by the temporal adjustment parameter, $f_t = 60$, is a singularity of this model, since, Nelson and Winter (1982) have considered a linear relationship to time.

Regarding imitation, there is a new stochastic event, which, in the case of success, results in the copy of the best market practice. The event's probability of success is given by:

$$P[d_{imt} = 1] = a_{im} \times RD_{jt} \tag{3}$$

Then, the probability of imitative success is proportional to the amount invested in R&D multiplied by a calibration parameter, which causes, regardless of the amount invested, the probability to be lower than 1. In accordance with Nelson and Winter (1982), this parameter is set to 1.25. The result of the imitation process is given by:

$$\hat{A}_{jt} = A_{jt} + d_{imt} \times (A^*_t - A_{jt}) \tag{4}$$

In case of failure of the probabilistic event ($d_{imt} = 0$) the productivity after the investment in imitation will be equal to the productivity of the previous period. In the case of success ($d_{imt} = 1$), the new value will be equal to the highest productivity among all firms, A^*_t.

In addition to these stochastic events, in the case of *genFirms*, there's stochasticity, in both genetic operators. The crossover procedure has the probability of occurring set to $P[X] = 0.7$, and each chromosome's gene mutation have a probability of happening equal to $P[M] = 0.03$.

Observation: the social welfare indicators are market price, average profit rate, capital and production concentration. The technical efficiency indicators: average and maximum productivities, market share, capital stock.

3.1.5. Initialization

Depending on the chosen market, two types of agents, *genFirms* and *nwFirms* or *genFirms* and *rndFirms*, produce a homogeneous product and initially have the same values of productivity, capital and market share. These agents have fixed or variable investment strategies and at the beginning of the first period, they make different investment decisions.

3.1.6. Input data

The environment is considered invariant over time, so there is no input data.

3.1.7. Submodels

Each firm produces the same homogeneous product following the production equation:

$$Q_j = A_j \times K_j \tag{5}$$

Then, the net profit rate of each firm (π_j) is given by:

$$\pi_j = pA_j - c - r_{im} - r_{in} \tag{6}$$

where the cost of capital (c), pre-set constant, and p is the product's market price, defined by the following equations:

$$Q = \sum_j Q_j \tag{7}$$

$$p = p(Q) = \frac{D}{Q^{1/\eta}} \tag{8}$$

where Q is the total offer, D is the demand, constant and predefined as 67, and η, is the elasticity of demand, considered according to the original model and equal to 1. After setting the price for the market, it is possible to compute, the profit rate, π_j that, after multiplying by the company's capital stock, K_J, results in the net profit of each firm, Π_j:

$$\Pi_j = \pi_j \times K_j \tag{9}$$

The effective productivity of the firms, for the following period, will be given by selecting the highest productivity value resulting from stochastic events:

$$A_{j,t+1} = \max\left(A_{jt}, \tilde{A}_{jt}, \hat{A}_{jt}\right) \tag{10}$$

So, the productivity of the previous period is compared to the ones obtained through innovation and after the imitation process, selecting the highest of them as the effective productivity for the next period.

Firms with fixed strategies use a fixed percentage of their capital as investment in innovation and imitation. According to Nelson and Winter (1982), in the case of a market with 32 firms these values are 0.00097 and 0.0194, respectively. These values were chosen, according to the authors, because they correspond to an average of two innovative successes per year and one imitation success was as likely for the whole industry as to one firm thriving in innovation. Therefore, the value invested in innovative R&D by the firms, will be equal to the highest rate of investment in innovation multiplied by the firms' capital stock:

$$RD_{in_{jt}} = r_{in} \times K_j \tag{11}$$

The firms' capital stock after each period, is determined by:

$$K_{j(t+1)} = I\left(\frac{P_t \times A_{j(t+1)}}{c}, \frac{Q_{it}}{Q_t}, \pi_{jt}, \frac{\delta}{f_t}\right) \times K_{jt} + \left(1 - \frac{\delta}{f_t}\right) \times K_{jt} \tag{12}$$

where I is the investment equation, a function of the ratio between price and production cost, $\frac{P_t \times A_{j(t+1)}}{c}$; market share, $\frac{Q_{it}}{Q_T}$; the firm's profit in the previous period, π_{jt}; and the adjusted depreciation rate, $\frac{\delta}{ft}$; which corresponds to the depreciation rate in the trimester, defined as 0.03 by Nelson and Winter (1982), divided by the time factor adjustment for the 6000 periods set as 60. The investment is defined in a way that is always non-negative:

$$I\left(\rho, s, \pi, \delta_t\right) = \max\left[0, \min\left[(1 + \delta_t) - \frac{(2 - s)}{\rho * (2 - 2s)}, f(\pi)\right]\right] \tag{13}$$

Being $f(\pi)$ determined according to the availability of external investment ($BANK$):

$$f(\pi) = \begin{cases} \delta_t + \pi & \text{if} \quad f(\pi) \leq 0 \quad or \quad BANK = 0 \\ \delta_t + 2\pi & \text{if} \quad f(\pi) > 0 \quad and \quad BANK = 1 \\ \delta_t + 3.5\pi & \text{if} \quad f(\pi) > 0 \quad and \quad BANK = 2.5 \end{cases} \tag{14}$$

The genetic algorithm: A flowchart of the general procedure of the genetic algorithm is shown in Fig. 1. First, a predefined number of agents is generated. The aforementioned agents have a fixed number of strategies randomly generated with a constant number of genes. These genes are the elements of a binary string. The agents test their chromosomes and keep information about their performance and, at the end of each generation, the best chromosome is selected to form the next generation. The other new chromosomes will be generated through crossover and mutation operators.

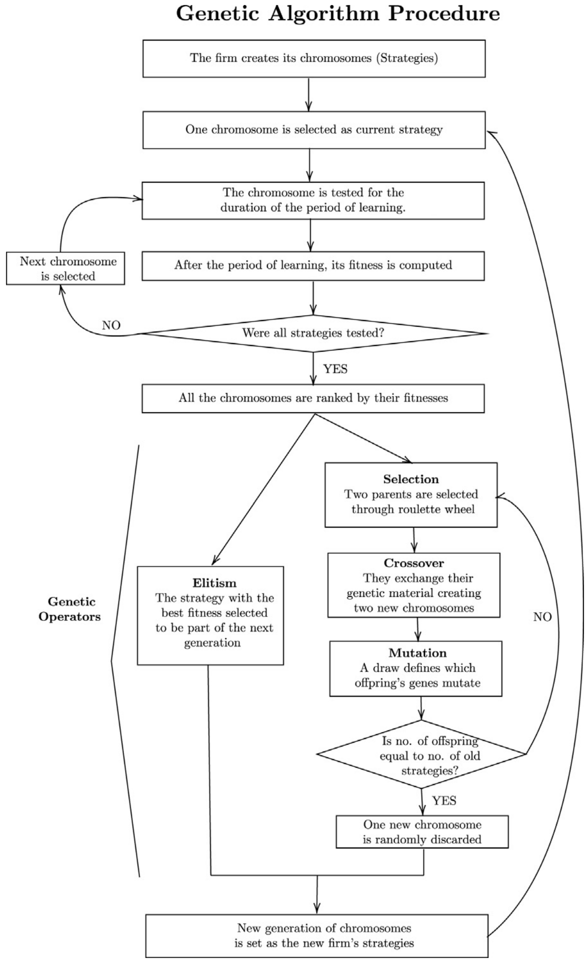


Fig. 1. Flow chart of the genetic algorithm process for generating new strategies.

As presented by Yildizoglu (2002), there are *Cr* strategies, defined as equal to 8 and each of these chromosomes has *G* genes. Therefore, for *G* equal to 7 the set of possibilities is determined by:

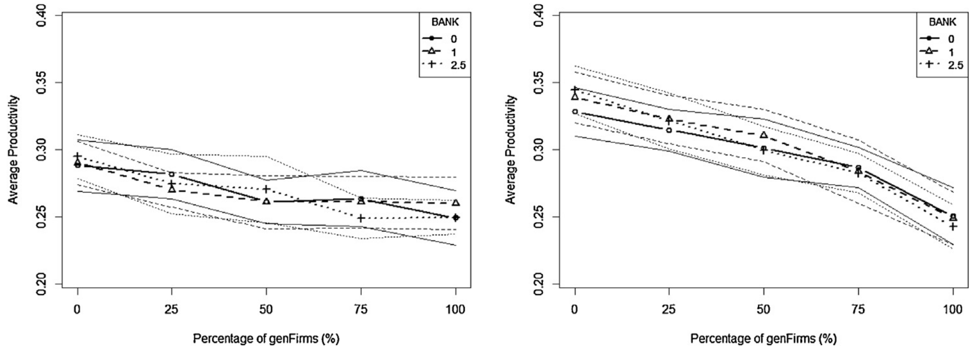
$$\Delta = \sum_{i=0}^{G-1} 1 \cdot 2^i = 127 \tag{15}$$

The first generation is created by randomly generating all the chromosomes, which represents a percentage of investment in innovative R&D. Therefore, considering the maximum possible investment in innovative R&D, with a 7-gene chromosome, there will be 127 possible equally spaced levels of investment. For example, one chromosome [1 0 0 1 0 1 1], the investment in innovation would be equal to 59% of the maximum possible innovative R&D investment, as shown below:

$$r_{in} = \frac{1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0}{127} \tag{16}$$

$$r_{in} = \frac{75}{127} = 0.59 = 59\% \tag{17}$$

The profit rate is determined through the balance between productivity and investment in capital stock, the two crucial factors for determining the profit rate. The process is repeated for each of the chromosomes for some periods (*learning period*) and the average profit rate is computed.



(a) *nwFirms* vs *genFirms* (b) *rndFirms* vs *genFirms*

Fig. 2. Average productivity for different markets, availability of credit and proportions of *genFirms*. (Two auxiliary lines of plus and minus one standard deviation were added for each data set represented on the figures. This pattern is repeated throughout the paper.)

The genetic algorithm applied has three stages: elitism, in which the best strategy is maintained for the next period without modification, crossover and mutation.

The crossover consists of selecting two parents, which, through the roulette wheel method, create the remaining 7 chromosomes. The two chromosomes selected as parents reproduce with a probability, $P[X=1]$ equal to 70%. And, if this stochastic event is successful, the single point reproduction will occur, i.e., from a chosen position of the binary string, there is the exchange of genetic material, and two new chromosomes are created, which will form the new list of strategies. When the crossover does not occur, the parents themselves are added to the set of strategies. In the case of 8 chromosomes, this process is repeated four times, generating 8 new strings, then one is eliminated, and the winning strategy of the previous period is added to the set (elitism). This way, the dimension of the chromosomes set is maintained.

The mutation stage consists of another stochastic process for each gene of the chromosomes that has gone through de crossover phase. The probability of mutation, $P[M=1]$, is equal to 3%. Then, in case of a successful draw, the corresponding gene will mutate, so if its value was 0, it becomes 1, and vice versa. The mutation of each gene is independent, and the winning strategy does not suffer mutation, because, in theory, it already has the most desired characteristics.

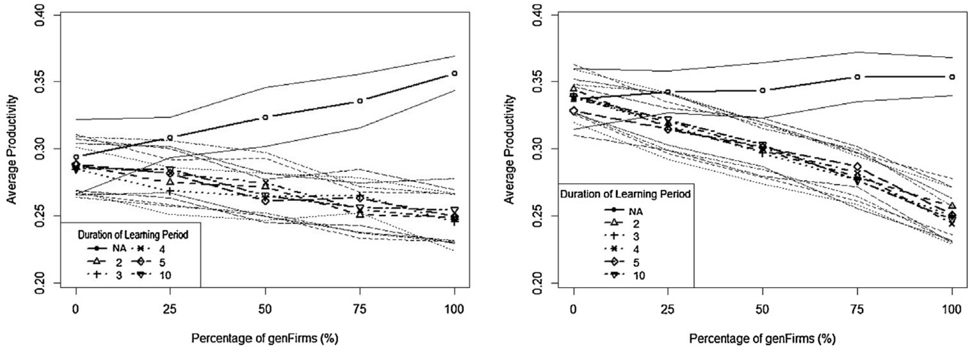
After the occurrence of crossover and mutation operators, the new chromosomes are, finally, selected for the next generation and all procedures are iteratively repeated until simulation ends.

4. Results and discussion

The average productivity is analysed at the end of both market simulations for different configurations considering, firstly, the effect of external investment by credit in market performance, Fig. 2, in this situation the number of learning periods was kept constant and equal to 5.

Thus, similar average productivity levels for the two markets are observed when the percentage of the companies using the genetic algorithm is 100%, but for smaller percentages, in a market that contains firms that use the fixed investment strategies, Fig. 2a, the average productivity levels are lower than in the market that contains firms, which use randomness in the decision, Fig. 2b. This situation is due to the fact that the percentage of capital that will be invested in innovation by the *rndFirms* will, on average, be greater than the *nwFirms*', since they can invest up to 5 times more in innovation than a *nwFirm*. This justifies the higher productivity levels for higher percentages of *rndFirms*. In both cases there is a decrease in average productivity as the ratio of *genFirms* increases and it can be observed that the availability of external credit does not lead to changes in the average productivity levels, a fact that corroborates the results shown in Nelson and Winter (1982).

When the average productivity is analysed in comparison with the percentage of *genFirms* and the number of learning periods, keeping the bank credit fixed at 0, it is observed that, as shown in Fig. 3, the average productivity behaves differently depending on the number of learning periods and the market involved.



(a) *nwFirms* vs *genFirms* (b) *rndFirms* vs *genFirms*

Fig. 3. Average productivity for different markets, learning periods and proportions of *genFirms*.

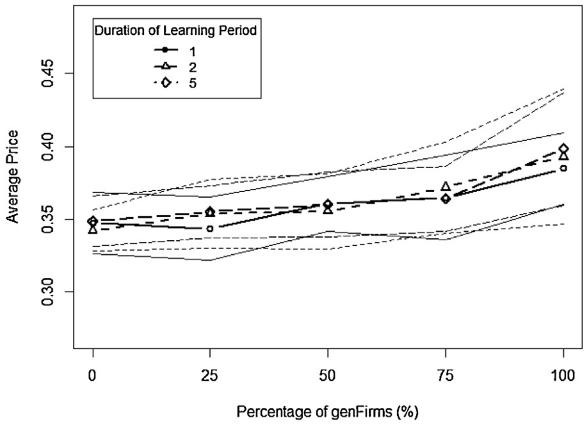


Fig. 4. Average price for different markets, learning periods and proportions of *genFirms*.

In a market with *nwFirms* and *genFirms*, Fig. 3a, when there is no learning process, an increase of the percentage of *genFirms* causes an increase in average productivity, but as the duration of the learning period is increased, the average productivity decreases as the ratio of *genFirms* increases.

Analysing the market with *rndFirms* and *genFirms*, Fig. 3b, where there is no learning, the average level of productivity is almost constant for all percentages of companies that use the genetic algorithm. For all the levels of learning, the model behaves similarly to the other market, with the productivity decreasing as the proportion of *genFirms* increases.

The non-change in average productivity alongside the increase in the ratio of *genFirms* in the markets shared with *rndFirms*, is an indication that for the minimal learning period, *genFirms* behave as firms that randomly choose their strategies. This statement is strengthened by the fact that in markets with *nwFirms* the average productivity increases as the proportion of *genFirms* is increased only for the situation where there is no learning process involved. In situations where learning is applied, in all the cases the behaviour of the average productivity does not change as the learning period is increased, in fact, there is an overlap of the averages and, respectively, standard deviations.

Considering the price of the products in both markets, there is no significant variation when changes in the duration of learning periods are considered, Fig. 4, or in the availability of credit. It is observed that the final price average and their respective standard deviations do not significantly modify their behaviours during the three different periods.

Comparing both markets' average prices, considering the situation where the learning period is equal to 5 and the bank credit is 0, Fig. 5, it is verified that the final average price is higher for the market containing fixed-strategy firms, (*nwFirms*).

The same situation is verified in all the proportions of firms using the genetic algorithm, with the exception of the case of 100% of *genFirms* (identical markets).

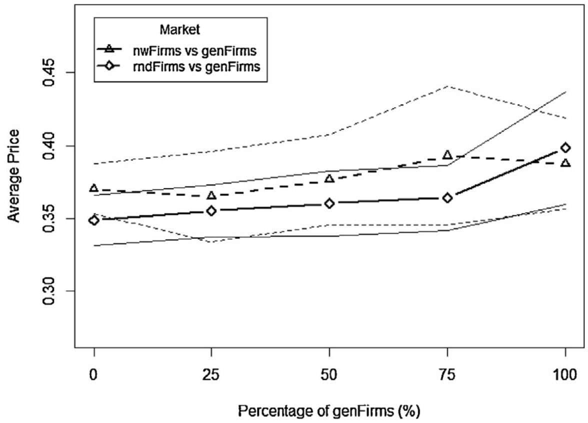


Fig. 5. Average price for different markets and proportions of *genFirms*.

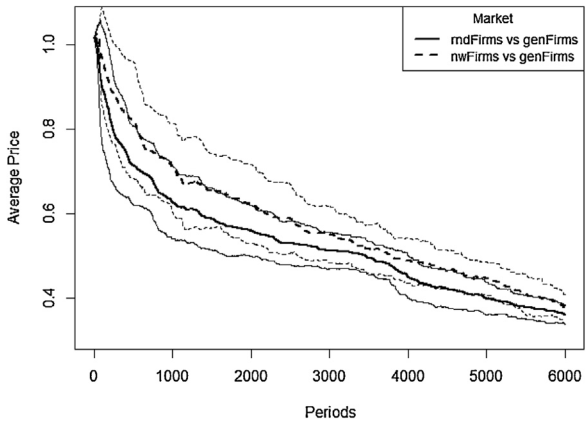


Fig. 6. Average price evolution through for both markets.

The highest average price in markets where *nwFirms* are present, indicates that, in these cases, the social welfare of the market is lower, because the higher average price decreases the consumers' purchasing power. On the other hand, in both markets, the increase in the percentage of *genFirms* seems to reduce the social welfare, because it indicates the increase in the average price. This finding goes against one of the results presented by Yildizoglu (2002), which indicated an increase in social welfare due to the presence of firms using the genetic algorithms, since, in that case, the price of the product decreased as the proportion of the firms using the algorithm increased.

In Fig. 6, it is possible to observe the evolution of the average price throughout the simulation, for both markets with a percentage of *genFirms* equal to 50%, learning period equal to 5 and zero credit. It is observed that, throughout the simulation the average price in q situation where *nwFirms* are present remains higher than in those where they are not. Probably, this is consequence of higher average productivity values due to a larger investment in innovative R&D by the rndFirms.

As expected, when analysing the investment in innovative R&D in the markets in the same situation described in the preceding paragraph, Fig. 7, it is observed that investment in the case of *rndFirms* and *genFirms* markets is much greater than the investment of the market that has *nwFirms*, this way, it is predictable that the first one will have greater success in innovation and achieve higher levels of productivity, which leads to lower average prices and, consequently, greater social welfare.

Analysing the average Inverse Herfindahl indexes for concentration of capital(*HK*) and of production(*HQ*), Fig. 8, there is a similar behaviour in the case of both markets, considering 50% of genetic firms, learning period equal to 5 and zero bank credit.

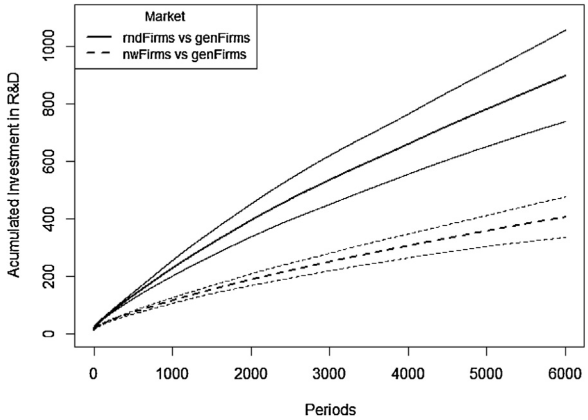


Fig. 7. Acumulated investment in R&D for both markets.

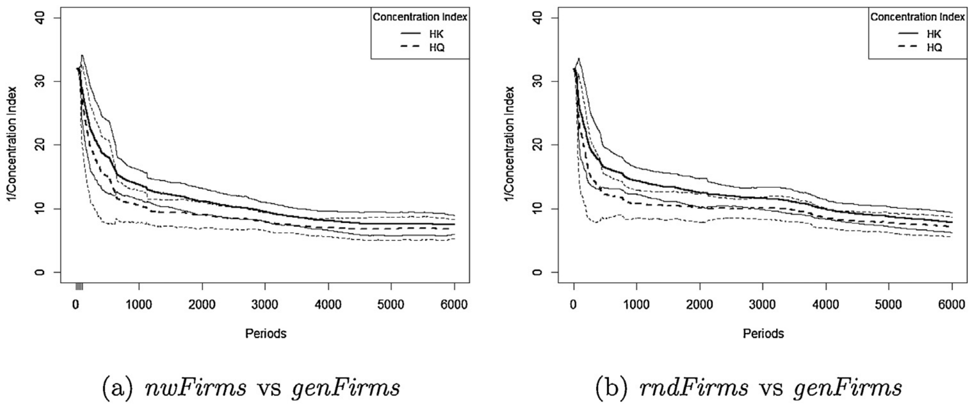


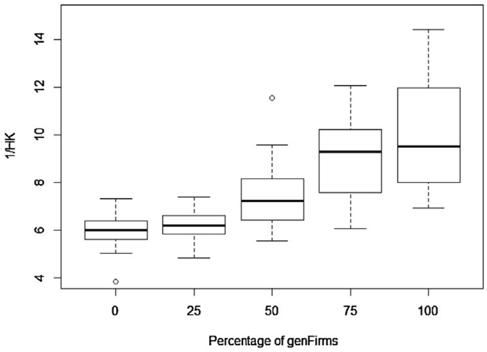
Fig. 8. Evolution over time of the production and capital Inverse Herfindahl indexes.

The concentration rates at the end of the simulation are then compared to each of the percentages of genetic firms considering the two markets, Fig. 9. It is possible to verify that both the production and capital indexes increase as the proportion of the companies using the genetic algorithm increases, which means that the market becomes less concentrated as firms that use algorithm are added. It turns out that both concentration indexes, when the ratio is 0% of *genFirms* and 25% of *genFirms*, for the market containing *nwFirms*, Fig. 9a and c, are smaller than in the market containing *rndFirms*, Fig. 9b and d. Therefore, it is concluded that the market containing *nwFirms* is generally more concentrated for smaller proportions of firms that use the genetic algorithms. In the case of higher proportions, this difference does not occur, being the indexes similar.

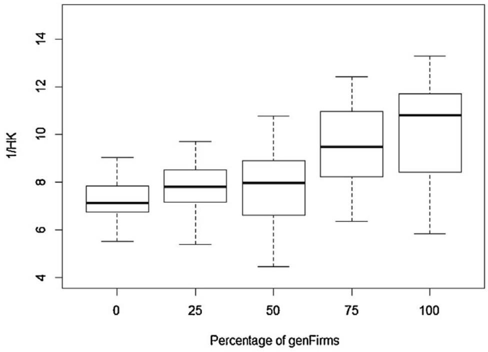
Analysing the results of net profit, Fig. 10, it is possible to verify that in the market containing genetic and fixed strategies' firms, both the average, Fig. 10a, and the maximum profit levels are superior when compared to the universe containing firms with random strategies. This fact is explained by the higher productivity levels of the *rndFirms*, which probably caused a reduction in the profit of *genFirms*. It should be noted that the shock, near the period 4000, Fig. 10a, indicates a success in innovation that modified the market structure.

It is essential to carry out the study about the convergence of the values selected by the genetic algorithm. It is verified that, for all proportions of genFirms, learning periods, bank credit availability, the behaviour of the *genFirms* is similar. Fig. 11, illustrates the convergence of the strategies' average values to 0; in the situation where 50% of firms that used the genetic algorithm, a learning period was set at 5 and zero bank credit.

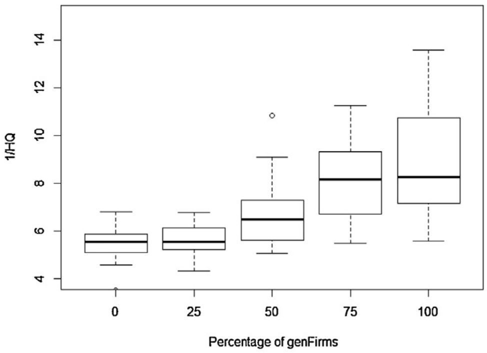
It turns out that the genetic algorithm has identified as best strategy the zero investment in innovation. This convergence is indeed curious, since firms using the genetic algorithm tended, then, to have the same behaviour as the imitative firms in the Nelson and Winter (1982) model. In that model, the imitative firms do not invest in innovative R&D, limiting themselves to imitate and invest in physical capital.



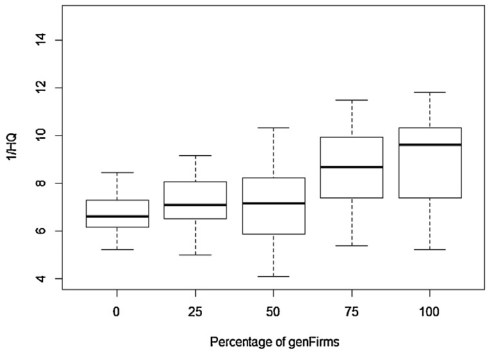
(a) Capital index for *nwFirms* vs *genFirms*



(b) Capital index for *rndFirms* vs *genFirms*

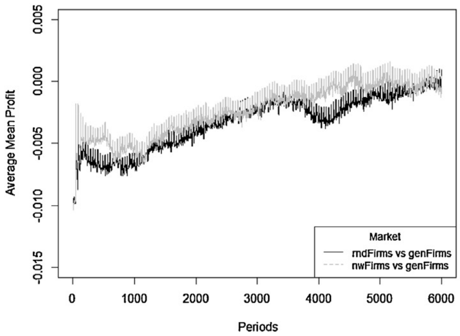


(c) Production index for *nwFirms* vs *genFirms*

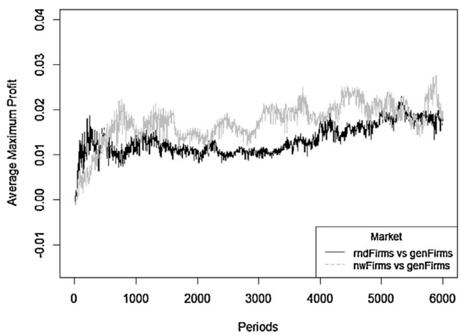


(d) Production index for *rndFirms* vs *genFirms*

Fig. 9. Capital and production Inverse Herfindahl indexes for different markets and proportions of *genFirms.*



(a) Average mean profit for *genFirms*.



(b) Average maximum profit for *genFirms*.

Fig. 10. Evolution of average mean and maximum profits of *genFirms* for different markets.

The firms that use genetic algorithm converge to this same situation. It was necessary to verify if this set up created a competitive advantage translated into market dominance. Then, it was necessary to evaluate the market share for each of the simulation universes across different genetic firms percentages, Fig. 12.

It is possible to conclude that for all analysed situations the *genFirms* dominate the market, because their market share always exceeds their proportion in relation to the other firms. It is clear that when the genetic firms divide

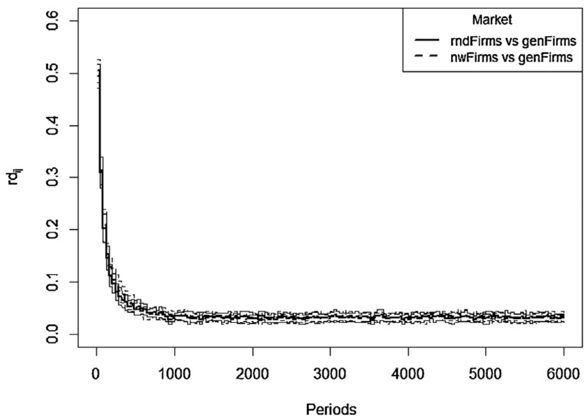


Fig. 11. Convergence of strategies for *genFirms* over time.

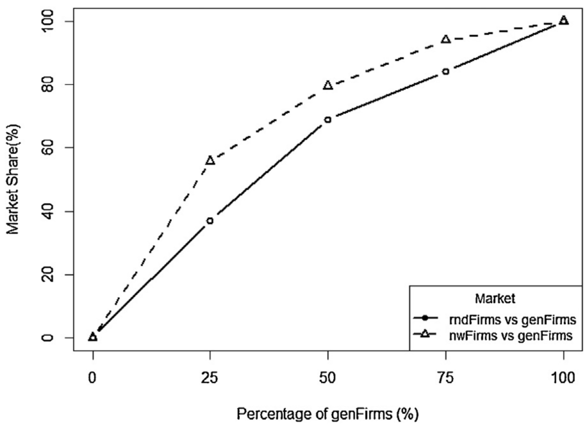


Fig. 12. Average market share of *genfirms* for different proportions of firms using the genetic algorithm.

Table 3
Market share of *genFirms*.

Market	Percentage of firms that use the genetic algorithm (%)				
	0%	25%	50%	75%	100%
nwFirms vs *genFirms*	0.00	55.75	79.42	94.00	100
(Standard deviation)	(0.00)	(18.95)	(17.65)	(6.11)	(0.00)
rndFirms vs *genFirms*	0.00	37.07	68.90	84.15	100
(Standard deviation)	(0.00)	(20.43)	(12.72)	(10.58)	(0.00)

the market with the fixed strategy firms, they have more advantage than when they coexist with those with random strategies. This fact is due to the higher productivity levels of these firms *rndFirms*, but, even in this case, the firms using the genetic algorithm outweigh their proportions as demonstrated by Table 3:

The results confirmed that, when firms use this genetic algorithm, in most cases, they dominate the market, demonstrating that the technique used by them is a winning strategy, sharing Yildizoglu (2002) findings and confirming the existence of individual learning. The results have shown a different convergence than the one found by Yildizoglu (2002). The genetic firms tended throughout the simulation to the purely imitative firms of the Nelson and Winter (1982) evolutionary model. This convergence, combined with the investments in innovation made in the initial periods, resulted in a strategy that dominates the market.

5. Conclusions

The Nelson and Winter (1982) evolutionary model is widely used as a basis study and allows numerous forms of adaptations. In one of those, Yildizoglu (2002) implemented a simplification of said model, using a genetic algorithm as a way to govern the firms' R&D decisions. Among the conclusions the author has shown a proof of learning by the firms that use the genetic algorithm, as result of the market dominance and the convergence of the individual strategies.

The creation of a new model that sought to be most faithful to the original model, Nelson and Winter (1982), was proposed. Using some modifications proposed by the simplified model, Yildizoglu (2002), and other new assumptions.

The number of firms used in the simulations was identical to the competitive market of the original model, as well as their investment rates in innovative and imitative R&D. The number of periods followed the one used in the simplified model, so it was decided to make a temporal adjustment in the depreciation rate of each period and in the stochastic events that determine the successes in innovation and imitation. It was decided not to implement the exit of firms from the market. The maximum investment possible of firms with variable strategies was set to five times the percentage of the capital stock invested by the firms with fixed strategies of the original model. The investment is also another parameter that has been modified in relation to the work of Yildizoglu (2002), the author used as reference the previous period profit, while in the initial work, it was used, as in the present study, the percentage of capital stock.

The model was described using the "ODD protocol" proposed by Railsback and Grimm (2012). Then, the model was implemented, and the simulations were performed in Netlogo using two distinct models, the first of them contained firms that used fixed strategies of investment in innovative R&D along with firms that used a genetic algorithm, and, the second model simulated firms with random decision strategies on investment in innovation together with the same firms that used the algorithm. Different proportions of genetic firms were tested, three credit availability, and six durations for learning periods, seeking to verify the effects of each of those variables in the market structure.

In relation to the availability of external credit, the results corroborate the findings of Nelson and Winter (1982). It was verified that there was no direct influence of the increase in credit availability over the productivity or significantly changes the market structure.

The analysis of the learning period duration has shown that when learning processes are not applied there is not sufficient selectivity of the strategies by the firms that used the genetic algorithm. So, as these strategies did not show any convergence, the firms ended up presenting a similar behaviour, as it was possible to predict, and performance when compared to the random strategies ones. In learning periods higher or equal to two intervals, a convergence was found, demonstrating that for this genetic algorithm and in these cases, specifically, "short" learning periods were sufficient to offer the necessary selectivity for the strategies evolutionary process.

In both models the presence of genetic firms decreased the average productivity levels, leading to the increase in the average price in the markets. Therefore, the presence of firms using the genetic algorithm causes a decrease in social welfare, as increases in price reduce the purchasing power of consumers. This result opposes the one presented by Yildizoglu (2002), which verified a decrease in price and consequent increase of the social welfare.

During all the simulations, the fall of the average price was verified, which, in the end, was in the same range of those found by Nelson and Winter (1982), indicating the model suitability. Considering the investment in innovation, it was found, as expected, that markets with random firms invested more than those that contained similar market divisions, but with firms that used fixed strategies. This is explained by the fact that companies with random strategies have a limit of the investment rate five times higher than firms with fixed strategies, therefore, on average, the innovative investment rate will be higher, explaining the largest productivity levels for markets with random decision firms.

The concentration indicators have shown that for both markets, higher percentages of firms that govern their innovative investment using the genetic algorithm led to a less concentrated market. Therefore, capital and production are more equally distributed, and could even mean a greater competitiveness of the market, however, this hypothesis is not confirmed by lower price levels.

The analysis of the genetic firms' strategies convergence found a result partially similar to the one presented by Yildizoglu (2002), because there was a convergence, but unlike the previous work, investment in innovation has converged to levels close to zero. This fact is curious, because the behaviour of the genetic firms over time became a similar to the, Nelson and Winter (1982), imitative firms, which do not invest in innovation.

Analysing the effects of reducing the investments in innovative R&D, showed that this was a dominant strategy, since the genetic firms' market share have always outweighed the percentage equivalent to their presence in the market.

So, they manage to conquer an extra portion of the market. This effect is more strongly observed in the situation where the genetic firms share the market with the fixed strategies ones and for small percentages of firms that make decisions using a genetic algorithm.

As it was stated by Yildizoglu (2002), firms using the genetic algorithm learn, because even acting independently, their level of intelligence allows them to adjust their strategies in order to possess market dominance, showing a convergence of strategies, clearly proofing the existence of an individual learning process.

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