Robust feeder reconfiguration in radial distribution networks

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ABSTRACT

Distribution feeder reconfiguration has been an active field of research for many years. Some recent theoretical studies have highlighted the importance of smart reconfiguration for the operating conditions of such radial networks. In general, this problem has been tackled using a multi-objective formulation with simplified assumptions, in which the uncertainties related to network components have been neglected by both mathematical models and solution techniques. These simplifications guide searches to apparent optima that may not perform optimally under realistic conditions. To circumvent this problem, we propose a method capable of performing interval computations and consider seasonal variability in load demands to identify robust configurations, which are those that have the best performance in the worst case scenario. Our proposal, named the Interval Multi-objective Evolutionary Algorithm for Distribution Feeder Reconfiguration (IMOEA-DFR), uses interval analysis to perform configuration assessment by considering the uncertainties in the power demanded by customers. Simulations performed in three cases on a 70-busbar system demonstrated the effectiveness of the IMOEA-DFR, which obtained robust configurations that are capable to keep such system working under significant load variations. Moreover, our approach achieved stable configurations that remained feasible over long periods of time not requiring additional reconfigurations. Our results reinforce the need to include load uncertainties when analyzing DFR under realistic conditions.

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1. Introduction

Reconfiguration of radial distribution networks is a viable option for ensuring optimal or nearly optimal operation in the presence of several constraints and variable power demand [7,16,18–22,35]. From a theoretical perspective, distribution networks can always be reconfigured whenever imminent loading variations are detected. Even though optimization procedures may reveal improvements on power losses, voltage flatness or load balancing related to a given distribution network, it is important to realize that maneuvers performed by power utilities are still costly, lack appropriate equipment [35], and hence, they are unusual when such systems are operating normally. Although feeder reconfiguration seems to be very attractive to optimize the system's operating conditions, supervisory centers adopt a conservative strategy to avoid switching maneuvers as much as possible. This behavior can be justified by the low level of network automation and the

imminent risks due to interference with protective devices, which may cause unexpected overloads or interruptions in the power supply. From the practical standpoint, maneuvers are the preferred procedure for solving critical or abnormal situations, but rarely for performance optimization.

Under normal conditions, seasonal variations in power demand require utility companies to rely on demand estimation models to properly meet demand within safety margins and quality indexes [4,6,16]. In these models, consumers are statistically classified according to their average demand [6]. Loading forecasts contain inherent errors because these approaches are based entirely on estimations. In addition, renewable energy has given its contribution to the power demand variation on load buses which originally acted as strict consumers. Its importance to the distribution systems may no longer to be neglected once the growing proportion of power generated by alternative means is intended to fill the gap between peak load demands and energy availability at regional extent [7].

A naive approach would treat network parameters as exact or definite values, and this approach could mistakenly assess a given configuration as optimal, adequate or fair to uncertainty-free environments. However, these configurations may be insufficient or even impracticable in the presence of power demand uncertainty.

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Nomenclature

N_B	bus set size	$ X $	size of a given set X
N_L	distribution line set size	$ v $	modulus of the variable v
N_s	number of sources	$\Re$	set of real intervals
N_w	total number of switches	$[a]$	interval number
N_f	number of objective functions	$[a]$	interval vector
N_v	number of uncertainty parameters	a^+	upper bound of an interval vector
$\mathbf{x}$	binary vector containing all the switches statuses	$\mathbf{P}$	set of uncertainties
$\mathbf{x}^c$	c th configuration	$\mathbf{X}$	set of decision variables
$\mathbf{B}^s$	set of all buses supplied by source s	$\mathbf{p}$	vector of uncertainty parameters
Z_i	impedance of the i th distribution line	$\mathbf{f}$	vector of objective functions
I_i	current flow in the i th distribution line	$\mathbf{g}$	vector of inequality constraints
$I_i^{\max}$	ampacity (maximum current) of the i th distribution line	$\mathbf{h}$	vector of equality constraints
V_k	actual voltage at bus k		
V_k^{nom}	nominal or normalized (p.u.) voltage at bus k		

Hence, it is important to obtain robust solutions. In our approach, robust solutions are those solutions that are less susceptible to uncertainties and they are able to maintain a system normally operating even in the worst case scenario. In this work, we restricted uncertainties to PQ loads, because they are among the most relevant factors in feeder reconfiguration decisions, mainly if we consider power generation in those buses [12,32–34]. Because feeder reconfiguration depends on some power flow method [2], uncertainties must be accounted for using a mathematical model capable of reflecting all immediate effects on the power flow [11,12,32,33].

Feeder reconfiguration is considered a combinatorial problem with an irregular search space [26,31] and is generally formulated as a restricted multi-objective problem. The radiality constraint and the discrete nature of the switches prevent the use of classical techniques to solve feeder reconfiguration [2,18,19,23,25,27]. An evolutionary stochastic approach was chosen because this approach is relatively unaffected by the nature of the problem. As previously mentioned, an efficient solution set in a deterministic environment can be partially or totally impaired in a perturbed environment, which moves these solutions away from the Pareto frontier [14]. Thus, a conventional approach will probably fail to identify an efficient solution set to the feeder reconfiguration problem. Therefore, we present a proper implementation, in which a Multi-Objective Evolutionary Algorithm (MOEA) based on NSGA-II [13] was coupled to an interval version of the Backward/Forward Sweep Method (BFSM). This novel hybrid approach, which we call Interval Multi-objective Evolutionary Algorithm for Distribution Feeder Reconfiguration (IMOEA-DFR), is a robust method of solving

reconfiguration problems. To our knowledge, there is no study encompassing this specific issue yet. The objectives of this work are (i) to provide a new strategy that properly addresses load uncertainties in DFR optimization (see Section 5), (ii) to demonstrate the fragility of optimal solutions found by conventional approaches in uncertain real-world environments (see case 2 in Section 6), and (iii) to discuss how robust solutions can replace pseudo-optimal solutions to ensure reasonable and satisfactory network operating conditions in the worst case scenario (see case 3 in Section 6).

This paper is organized as follows. In Section 2, related works are cited, and their primary contributions are highlighted. The formulation of the DFR problem is presented in Section 3. In Section 4, the background of interval analysis is addressed, and the robust version of the DFR problem is defined. The implemented algorithm and related issues are described in Section 5. Finally, a discussion about computational experiments is presented in Section 6, and concluding remarks are given in Section 7.

2. Related works

The feeder reconfiguration has been treated as a typical uncertainty-free problem in many papers [2,3,7–10,15,25,26,29]. From a classical perspective, these approaches are satisfactory, and their solution sets are valid whenever uncertainties are negligible. However, these solutions are not as reliable as they seem [5]. It has been shown that electrical power systems are subjected to many sources of uncertainty such as power (load) demand [16,20–22], distributed generation [21,22], and electrical parameters [5,6,35].

Fig. 1. Computation of the worst case performance for some solutions in the decision variable space – 2D example.

Fig. 2. (a) Crossover operator, (b) mutation operator, and (c) IMOEADFR flowchart.

Due to the importance of these uncertainties regardless their nature in the reconfiguration problem, seen in Section 1, it is noteworthy to mention the works that have addressed power flow methods under uncertain environments. Some authors have proposed versions of power flow analysis to cope with uncertainties [6,12,32,34,35]. Load variations were modeled for network planning purposes by Chaturvedi et al. [6] and later by Borges and Martins [4]. In [6], a proposal was devised based on interval arithmetic approach that represents loads through Gaussian probability density functions. Vaccaro presented an alternative to the sampling-based technique applied to radial power flow analysis supported by interval mathematics [32] or affine arithmetic [33]. He was the first to introduce an affine arithmetic approach [33] to find the distribution network's operating point, while considering variation in load and generation levels. His proposals were intended to tighten variables' domains without losing valid solutions. Still, fewer loadflow methods are able to deal with uncertainties in radial networks [4,6,11,27]. Das used an interval analysis technique to perform power flow analysis in unbalanced [12] and balanced [11] three-phase radial distribution systems while considering load and feeder parameters uncertainties. Zhang et al. [35] proposed a method called Reliability-Oriented Reconfiguration (ROR) which employed interval analysis to quantify the impact of uncertain data on system parameters and to maximize the possi-

bility of reliability improvement and/or loss reduction achieved through reconfiguration.

As previously detailed, many researchers have studied strategies for coping with uncertainties in power flow analysis to tackle reconfiguration problem in a broader manner. This work goes farther and deals suitably with the robust DFR problem focusing on network operation. Our IMOEADFR approach applies interval analysis not only to power flow algorithms, but also to non-dominance assessments among solutions in terms of the worst case performance. Therefore, the primary contribution of this manuscript is to provide an effective method of finding robust configurations that guarantee the operating conditions of distribution feeders under uncertain loading.

3. Problem formulation

Our mathematical formulation is derived from [3]. The on/off statuses of all N_w maneuverable switches were used as optimization variables and coded into a binary vector $\mathbf{x} = [x_1, \dots, x_i, \dots, x_{N_w}]$ whose i th element indicates whether the corresponding switch is open ($x_i = 0$) or closed ($x_i = 1$). Each distribution line has its own switch ($N_L = N_w = \|\mathbf{x}\|$). As in [15], four objectives were assumed in this formulation: Power Losses (PL), Current Loading

Fig. 3. Test feeder with 70 busbar found in [2,9,15].

Index (CLI), Voltage Deviation Index (VDI), and Number of Switching Maneuver (NS). PL can be described as follows:

$$PL = \left(\sum_{i=1}^{N_w} x_i * Re(Z_i) * |I_i|^2 \right), \quad (1)$$

where $Re(Z_i)$ is the real component of the i th line impedance and $|I_i|$ corresponds to the magnitude of the current flowing through the i th line. The second objective is mathematically represented as:

$$CLI = \left(\frac{1}{N_w} \sum_{i=1}^{N_w} x_i * I_i / I_i^{\max} \right). \quad (2)$$

The third objective involves minimal voltage deviation from the nominal values at all N_B buses in the system. To implement this criterion, the index proposed by Sahoo et al. [25], which requires simple computations, was modified by Huang and represented as [15]:

$$VDI = \left(\frac{1}{N_B} \sum_{k=1}^{N_B} x_{i,k}^c * |V_k - V_k^{\text{nom}}| \right), \quad x_{i,k}^c \in \mathbf{W}, \quad (3)$$

where $\mathbf{W}$ denotes a set of all closed switches and $x_{i,k}^c$ indicates that switch i is closed to connect bus k to a power node in configuration c . The fourth objective considers the number of switching maneuvers, which must be kept as small as possible due to operational costs and network management considerations. This objective is calculated using the Manhattan distance:

$$NS = \sum_{i=1}^{N_w} |x_i^c - x_i^0|, \quad (4)$$

where x_i^c and x_i^0 denotes each set of corresponding switches in the candidate and previous configurations, respectively. The distance between any corresponding switch equals $|x_i^c - x_i^0|$. Additional, restrictions were admitted. Radiality must be ensured using the following relations:

$$N_B - N_s = \|\mathbf{W}\| = \sum_{i=1}^{N_w} x_i^c, \quad (5.a)$$

$$\sum_{s=1}^{N_s} b_k^s = 1, \quad \forall k = 1, \dots, N_B, \quad (5.b)$$

$$x_{i,k}^c \leq x_{i-1,k-1}^c |x_{i-1,k-1}^c = 1 \iff B^s := B^s \cup b_{k-1}^s, \quad \forall i, \quad i-1 = 2, \dots, N_w, \quad \forall k = 2, \dots, N_B. \quad (5.c)$$

The condition in Eq. (5.a) holds for any c radial configuration or set of configurations, but it does not guarantee radiality by itself. In Eq. (5.b), b_k^s is a binary variable that takes the value one (zero) to indicate that bus k is fed (not fed) by source s . This statement is complemented by Eq. (5.c), wherein the status of the i th switch ($x_{i,k}^c$), which delivers power to the k th bus, depends on the status of an adjacent switch, ($x_{i-1,k-1}^c$), closer to the (temporary) common source s , which in turn delivers power to the $k-1$ th bus. All of the N_B buses must be connected to available sources by a unique path to avoid loop or islanding. The following restriction ensures that the current in each line (or branch) does not exceed its ampacity:

$$|I_i| \leq I_i^{\max}, \quad i \in \mathbf{W}. \quad (6)$$

The final restriction establishes that no valid solution may violate upper and lower limits of the bus voltages:

$$V_k^{\min} \leq V_k \leq V_k^{\max}, \quad k = 1, \dots, N_B. \quad (7)$$

The values of $V_k^{\min}$ and $V_k^{\max}$ were assumed to be 0.9 p.u. and 1.05 p.u. on our simulations, respectively, according to the values defined in Baran's work [2]. It is worthy to note that voltage restriction is complementary to Eq. (3) whose goal is to reduce as much as possible bus voltage variation.

3.1. Component modeling issues

Power can be re-routed through reconfiguration, i.e., opening and closing sectionalizing and tie switches. Reconfiguration is an effective way to operate distribution networks under load demands changes by redistributing currents through various lines to prevent or alleviate overloading in system equipment, reduce real power losses, improve the voltage profile and increase system stability. Feeders may serve a variety of different consumers who can be organized into classes such as residential, commercial, or industrial [16]. Each class has its own peak time, and their load behaviors affect the overall loading over the 24-h cycle. Depending on the chosen configuration and distribution of those classes, restrictions can be violated in some instances. In these cases, the system must be reconfigured in advance to avoid potential risks to system stability (e.g., equipment overloading or non-prescribed bus voltages).

In one of the experiments described in Section 6, loads are modeled as power sinks with variations prescribed by demand curves [28]. Seasonal behavior is incorporated into the loads according to the five profiles found in [28]. These profiles were assigned to various loads according to their nominal values, which are specified in the original test feeder. Moreover, system parameters (e.g., line impedance and available power) are treated as discrete, error-free quantities. Each demand curve statistically describes a group of similar consumers with identical profiles and thus has

Fig. 4. Ranges of the (a) real and (b) reactive power demands of the load buses in case 1.

an inherent degree of uncertainty. We incorporate uncertainties by assigning specified ranges to each bus. Loads can vary considerably throughout the day. Thus, the current configuration may be suboptimal or even invalid due to violations of some restrictions. These variations can be understood as perturbations to the system. In this work, a perturbation is characterized by well-defined uncertainty ranges on specific buses. To account for uncertainties, real and reactive power loads at the buses were treated as interval numbers.

4. Interval analysis

This section addresses the basic concepts and definitions of Interval Analysis (IA) required to understand how the proposed algorithm deals with intervals. More detailed information can be found in [17]. IA approaches use a range-based model that represents a punctual number and its uncertainty. The model presented in this work uses interval numbers to indicate the spread of a numerical value. A real, closed and connected interval is defined as:

Definition 1. A closed interval $[a]$ is a connected subset in $\mathfrak{R}$ defined by

$$[a] = [a^-, a^+] := \{a \in \mathfrak{R} | a^- \leq a \leq a^+\}, \quad (8)$$

where a^- and a^+ represent the lower and upper bounds of $[a]$, respectively. The set of all intervals defined by (8) represents the set of real intervals ($\mathfrak{R}$).

Definition 2. The multi-dimensional real interval $[a] \in \mathfrak{R}^n$, denominated box or interval vector is given by

$$[a] = [a_1] \times [a_2] \times \dots \times [a_n]. \quad (9)$$

Definition 3. The lower bound a^- of $[a]$ is given by $a^- = (a_1^-, a_2^-, \dots, a_n^-)^T$. In the same way, the upper bound $a^+ = (a_1^+, a_2^+, \dots, a_n^+)^T$.

Definition 4. The four classical operations of real arithmetic can be extended to intervals. For any such binary operator, denoted by Θ , perform the operation means computing

$$[a] \Theta [b] := \{a \Theta b | a \in [a] \wedge b \in [b]\}. \quad (10)$$

Regarding the extreme bounds, each classical operation for non-empty closed interval is given by:

$$[a] + [b] = [a^- + b^-, a^+ + b^+]. \quad (11)$$

$$[a] - [b] = [a^- - b^+, a^+ - b^-]. \quad (12)$$

Fig. 5. Ranges of the (a) real and (b) reactive power demands of the load buses in case 2.

$$[a] * [b] = [\min(s), \max(s)], \text{ being } s = \{a^- * b^-, a^- * b^+, a^+ * b^-, a^+ * b^+\}. \quad (13)$$

$$[a]/[b] = [a] * (1/[b]),$$

Where

$$\begin{aligned} 1/[b] &= \phi, \text{ if } [b] = [0, 0]. \\ 1/[b] &= [1/b^+, 1/b^-], \text{ if } 0 \notin [b]. \\ 1/[b] &= [1/b^+, \infty], \text{ if } b^- = 0 \wedge b^+ > 0. \\ 1/[b] &= [-\infty, 1/b^-], \text{ if } b^- < 0 \wedge b^+ = 0. \\ 1/[b] &= [-\infty, \infty], \text{ if } b^- < 0 \wedge b^+ > 0. \end{aligned} \quad (14)$$

It is noteworthy that the interval addition, subtraction, multiplication, and division operators operate on any two interval operands to produce an interval result that contains all arithmetical combinations of the interval numbers describing the operands.

As shown in the next subsection, we represent the power demands on the PQ buses using interval numbers to guarantee that all uncertainties on the loads are taken into account. In terms of implementation, every real operation that uses interval variables is replaced by the respective interval operator in the power flow analysis algorithm. Therefore, the fundamental theorem of IA [17] guarantees that all obtained outputs are contained within the interval result.

IA includes an inherent pessimism related to the interval operations, meaning that the output interval may be overestimated. For

instance, the real expressions $f = x * (1 - x)$ and $g = x - x^2$ are equivalent in real arithmetic. However, assuming $[x] = [0, 1]$, their interval counterparts lead to distinct results given that $f[x] = [0, 1]$ and $g[x] = [-1, 1]$. Both results represent an enclosure of the correct output regarding the input interval. On the other hand, the former is narrower (is less pessimistic) than the latter. In general, this pessimism arises as a result of multiple occurrences of the same variable. Mathematical reorganization of the expression can resolve this problem, but variable multiplicity may not always be avoidable.

4.1. Non-dominance analysis in an interval context

In this paper, we compute the worst case performance of each solution by considering the uncertainty parameters (uncertainty in the load demand on each PQ bus) using intervals. The worst case performance of a configuration is computed using:

$$f_{wc}(\mathbf{x}, \mathbf{p}) = \max_{\mathbf{p} \in \mathbf{P}} f_i(\mathbf{x}, \mathbf{p}), \quad i = \{1, \dots, N_f\}. \quad (15)$$

In Fig. 1, three spaces representing the uncertainty, decision variable, and objective are represented in two dimensions to illustrate the procedure by which the objective functions are evaluated for each configuration in $\mathbf{X}$ by considering perturbations in $\mathbf{P}$. Thus, a configuration is associated with a box in the objective space. The

Fig. 6. Ranges of the (a) real and (b) reactive power demands of the load buses in case 3.

Table 1

Optimal solutions to the 70-busbar system (the last two data columns consider 8% uncertainty for all loads).

Parameter	Reference solution found in [15]	MOEA-DFR solution	Reference solution considering uncertainty effects	IMOE-DFR solution
Real power loss (kW)	113.406	113.406	[96.5678, 134.125]	[96.592, 134.1582]
Voltage deviation index	0.0129	0.0129	[0.011851, 0.013959]	[0.011804, 0.013905]
Average current loading index	0.1860	0.1860	[0.17297, 0.20342]	[0.17268, 0.20308]
Switching number	4	4	4	4
Overall minimal bus voltage (p.u.)	0.9288	0.9288	[0.92288, 0.93471]	[0.92288, 0.93471]
Maximum current loading index	0.9203	0.9203	[0.84664, 0.99388]	[0.84664, 0.99388]
Proposed solution (opened switches)	{13, 59, 70, 71, 74}	{13, 59, 70, 71, 74}	{13, 59, 70, 71, 74}	{14, 59, 70, 71, 74}

worst case performance is represented by the black filled-in circle on the upper right corner of each box. According to Definition 6, the dotted boxes are dominated by at least one of the other boxes.

Definition 5. Given that u and $v \in \mathcal{R}^{N_f}$, $u = (u_1, u_2, \dots, u_{N_f})$ dominates $v = (v_1, v_2, \dots, v_{N_f})$, denoted by $u < v$, if $f \forall i \in \{1, \dots, N_f\}$, $u_i \leq v_i \wedge \exists i \in \{1, \dots, N_f\} \mid u_i < v_i$.

Definition 6. The dominance operator is extended to interval computation as follows

$$[a] < [b] \iff a^+ < b^+. \quad (16)$$

4.2. The robust formulation

The worst case Robust Multi-objective Optimization Problem (RMOP) can be seen as maximizing the effects of uncertainty on the objective functions, and subsequently minimizing all objective functions, as described in [30]. Mathematically, given that $\mathbf{x} \in \mathbf{X} \subseteq \mathcal{R}^{N_w}$, $\mathbf{p} \in \mathbf{P} \subseteq \mathcal{R}^{N_p}$ and $\mathbf{f}(\mathbf{x}, \mathbf{p}) : \mathcal{R}^{N_w} \times \mathcal{R}^{N_p} \rightarrow \mathcal{R}^{N_f}$ the RMOP is defined by:

$$\min_{\mathbf{x} \in \mathbf{S}} \max_{\mathbf{p} \in \mathbf{P}} \mathbf{f}(\mathbf{x}, \mathbf{p}) = \{f_1(\mathbf{x}, \mathbf{p}), \dots, f_{N_f}(\mathbf{x}, \mathbf{p})\}, \quad (17)$$

where the feasible space is given by:

Table 2

The best frontier found by MOEA-DFR or IMOEA-DFR in three scenarios: (i) original scenario described in [15], (ii) the perturbed scenario, and (iii) the interval scenario.

Switching number	Original scenario (40 solutions)	Perturbed scenario (65 solutions)	Interval scenario (75 solutions)
0	1 ^a	1 ^a	1 ^a
2	1	11	3
4	4 (Huang solution included)	20 ^b	11 ^b
6	15	22	30
8	19	11	30

^a Initial solution.

^b Huang solution was discarded due to dominance criteria.

$$S := \{\mathbf{x} \in X | \mathbf{g}(\mathbf{x}, \mathbf{p}) \leq \mathbf{0} \wedge \mathbf{h}(\mathbf{x}, \mathbf{p}) = \mathbf{0} \forall \mathbf{p} \in \mathbf{P}\}. \quad (18)$$

Solving (17) allows us to find a set of robust minimizers, which can be written as:

$$X^* := \left\{ \mathbf{x}^* : \neg \exists \mathbf{x} \left[\max_{\mathbf{p} \in \mathbf{P}} f(\mathbf{x}, \mathbf{p}) < \max_{\mathbf{p} \in \mathbf{P}} f(\mathbf{x}^*, \mathbf{p}) \right] \forall \mathbf{x}^*, \mathbf{x} \in S \right\}. \quad (19)$$

Particularizing the RMOP to the DFR problem, we have $Nf = 4$ objective functions: f_1 = power losses, f_2 = current loading index, f_3 = voltage deviation index, and f_4 = number of switching maneuvers. In addition, there are three constraints: h_1 = involving radiality – formed by Eqs. 5.a, 5.b, 5.c, g_1 ensuring current values less than ampacity for each line (or branch) – Eq. (6), and g_2 which is related to the upper and lower limits of bus voltages – Eq. (7). The values of f_1 , f_2 , f_3 , and g_1 , g_2 are directly influenced by the uncertainties $\mathbf{p}$ existing in load demands of each PQ bus. Hence, for the first three objective functions and two inequality constraints, intervals are obtained as output of the IMOEA-DFR. Since f_4 is calculated by the number of switches to be maneuvered in order to change the initial configuration into the candidate solution, the influence of uncertainties is not straightly represented in this objective function. The constraint h_1 considers the topology of the configuration. Consequently, power demand uncertainties do not have an immediate effect over configuration topology. In practical terms, f_4 and h_1 can be represented by integers and, thus, they are handled as degenerated intervals in which lower and upper bounds assume the same value.

5. Implemented algorithm

As stated in Section 1, DFR is a discrete combinatorial problem, and classical techniques are thus inappropriate for solving this optimization problem [18,19,23], which is reinforced by the radiality constraint. Many other techniques proposed in the literature are based on heuristic search strategies. In addition to being time-consuming, these techniques are restricted to network specificities and do not guarantee an optimal solution. Evolutionary

Fig. 7. Real power loss for three configurations: original, Huang and IMOEA-DFR.

algorithms are potentially capable of finding optimal or near-optimal solutions because they are not as limited as heuristics algorithms. Here, we propose an evolutionary algorithm based on NSGA-II [13] as the framework for finding non-dominated solutions. The fitness evaluation is computationally costly, and every new solution must be submitted to the iterative Backward/Forward Sweep Method (BFSM), which employs the recursive sets of equations found in [9]. Both the P-Q decoupled method and the classical Newton–Raphson method are unsuited to solving distribution network power flow. For this reason, the BFSM method with power aggregation [1] was used to compute bus voltages and power losses.

The topological analysis, which generates and modifies configurations, is another costly task and demands a considerable fraction of the overall computing time. To assure the feasibility of all evaluated individuals including those generated and those obtained from crossover and mutation operators, a Prim-like constructive heuristics is employed. A straightforward analogy can be made between the distribution system and the graph theory. Buses and distribution lines are equivalent to nodes and edges. Any distribution network can be represented as a graph $G(N, A)$ that contains a set of nodes, N , and edges, A . We employ a constructive heuristic for two purposes: (i) to create random radial configurations, and (ii) to perform crossover between two radial configurations. This method is simple and easy to implement, and solution diversity is guaranteed because every edge has the same probability of being chosen. More details about the crossover procedure based on a Prim-like heuristic are given in Algorithm 1.

Table 3

Performance comparison between Huang configuration and solution found by IMOEA-DFR. The best objective values are in italics.

Configuration	Original scenario (nominal loads)					Perturbed scenario (loads with uncertainty)				
	Real power loss (kW)	Voltage deviation index	Average current loading index	Overall minimal voltage (p.u.)	Maximum current loading index	Real power loss (kW)	Voltage deviation index	Average current loading index	Overall minimal voltage (p.u.)	Maximum current loading index
Huang solution [13] {13, 59, 70, 71, 74}	113.406	0.0129	<i>0.1860</i>	0.9288	0.9203	73.219	0.0169	0.1759	0.9582	0.9159
Identified solution by IMOEA-DFR {13, 20, 59, 65, 70}	<i>107.962</i>	<i>0.0120</i>	0.1872	0.9322	0.9203	<i>55.902</i>	<i>0.0107</i>	<i>0.1752</i>	0.9745	0.9159

Table 4
Performance details of three configurations (O: original configuration, H: Huang configuration, and I: configuration identified by IMOEAD-DRF) at seven times during the day in terms of Real Power Loss (PL), Voltage Deviation Index (VDI), Current Loading Index (CLI), and Minimal Bus Voltage (MBV).

Index	Time of day (h)																				
	6-h		10-h		12-h		16-h		18-h		22-h		24-h								
	O	H	O	H	O	H	O	H	O	H	O	H	O	H	O	H	O	H			
PL	24.9296	13.4737	12.6627	134.2435	77.1615	72.3263	144.8284	83.4442	78.4205	86.2404	47.5188	44.5449	71.7563	37.8639	35.4880	42.6348	21.8797	20.4509	19.6122	10.3552	9.7072
VDI	0.0095	0.0047	0.0045	0.0200	0.0095	0.0089	0.0206	0.0097	0.0091	0.0167	0.0081	0.0075	0.0159	0.0077	0.0073	0.0131	0.0066	0.0062	0.0086	0.0043	0.0041
CLI	0.0832	0.0748	0.0743	0.1728	0.1452	0.1452	0.1773	0.1481	0.1480	0.1425	0.1224	0.1220	0.1343	0.1181	0.1173	0.1133	0.1043	0.1033	0.0749	0.0680	0.0675
MBV	0.9701	0.9788	0.9802	0.9272	0.9437	0.9467	0.9242	0.9413	0.9443	0.9425	0.9564	0.9589	0.9485	0.9624	0.9646	0.9623	0.9739	0.9758	0.9738	0.9818	0.9831

Algorithm 1. Constructive heuristic adapted for crossover.

FUNCTION: Generation of two feasible configurations.
INPUT: $C_1 = G(N, A_1)$, $C_2 = G(N, A_2)$ and $G(N, A)$ in which $(A_1, A_2 \subset A)$
OUTPUT: C'_1, C'_2

- 1 $n \leftarrow 1$
- 2 **While** $n \leq 2$
- 3 Define residual set $R := A \setminus \{A_1 \cup A_2\}$
- 4 Initialize the following sets: active nodes $O \leftarrow \phi$, included nodes $N' \leftarrow \phi$ and included edges $A' \leftarrow \phi$.
- 5 Select one node v and one of its incident edges $e = (v, u)$ at random from parent C_n .
- 6 $O \leftarrow \{v, u\}$ $N' \leftarrow \{v, u\}$ $A' \leftarrow \{e\}$ $A_n \leftarrow A_n \cup e$ ($v, u \in N \wedge e \in A_n$)
- 7 Let $i \leftarrow 3 - n$
- 8 **Do**
- 9 Choose node o ($o \in O$) randomly.
- 10 If $\exists e | e = (o, p) \in A_i \wedge p \notin N'$ then $sel = i$ and go to line 14,
- 11 Else if $\exists e | e = (o, p) \in A_{3-i} \wedge p \notin N'$ then $sel = 3 - i$ and go to line 14,
- 12 Else if $\exists e | e = (o, p) \in R \wedge p \notin N'$ then go to line 15,
- 13 Else $O \leftarrow O \setminus \{o\}$ and go to line 9.
- 14 $O \leftarrow p \cup O$ $N' \leftarrow p \cup N'$ $A' \leftarrow e \cup A'$ $A_{sel} \leftarrow A_{sel} \cup e$. Go to line 16.
- 15 $O \leftarrow p \cup O$ $N' \leftarrow p \cup N'$ $A' \leftarrow e \cup A'$ $R \leftarrow R \setminus e$
- 16 $i \leftarrow 3 - n$
- 17 **Until** $N \setminus N' \neq \phi$
- 18 $C'_n \leftarrow G(N', A')$
- 19 $n \leftarrow n + 1$
- 20 **End While**

The purely constructive heuristic builds a configuration from an initial node. Subsequent nodes are added one by one, according to the available edges. If a chosen edge creates a loop, it is discarded, and it is replaced by another, chosen at random. This procedure terminates when all nodes are included. A semi-destructive heuristic is used to repair a configuration when the mutation operator is applied. This heuristic is able to detect loops as well as islanding. An open edge is closed to form a loop, and another edge belonging to this loop is opened. Although time-consuming, these two heuristics prevent useless evaluations in the proposed evolutionary algorithm and enable efficient exploration of the entire search space. Fig. 2(a and b) shows how new candidate solutions are obtained from genetic operators.

We initially implemented a non-interval algorithm, called the Multi-objective Evolutionary Algorithm for Distribution Feeder Reconfiguration (MOEA-DFR), to validate our evolutionary strategy by comparing its results with those achieved by the Enhanced Genetic Algorithm [15] (see Section 6, case 1). The MOEA-DFR is incapable of dealing with uncertainties and uses non-interval BFSM and the dominance criteria based on Definition 5 in Section 4. Next, the MOEA-DFR was improved to include the ability to deal with the worst loading uncertainty case through the use of interval computation, resulting in the IMOEAD-DRF, which is illustrated in Fig. 2(c). All real mathematical operations and variables in the BFSM were replaced by their interval operator and interval variable counterparts, respectively. The algorithm only uses basic mathematical operations (addition, subtraction, multiplication and division) and two elementary functions (square root and exponentiation). In IMOEAD-DRF, the non-dominance criteria are implemented according to Definition 6 in Section 4.

Table 5

Hypervolume values for the results found by IMOEA-DFR.

Case number	Hypervolume metric for 10 runs										Indicators	
	1	2	3	4	5	6	7	8	9	10	Mean	Standard deviation
1	0.0355	0.0349	0.0349	0.0341	0.0348	0.0357	0.0353	0.0352	0.0351	0.0349	0.0351	0.0004
2	0.0214	0.0205	0.0212	0.0207	0.0195	0.0212	0.0212	0.0207	0.0209	0.0213	0.0208	0.0006
3	0.0351	0.0384	0.0379	0.0377	0.0351	0.0354	0.0350	0.0352	0.0379	0.0381	0.0366	0.0015

Table 6

IGD values for the results found by IMOEA-DFR.

Case number	Inverse generational distance for 10 runs										Indicators	
	1	2	3	4	5	6	7	8	9	10	Mean	Standard deviation
1	0.1365	0.2109	0.3338	0.1518	0.2046	0.1018	0.2264	0.0526	0.1514	0.2213	0.1791	0.08
2	0.8624	3.3745	0.5525	1.0353	1.2171	0.6788	0.7022	0.9399	0.9791	1.0921	1.1434	0.81
3	0.2537	0.1944	0.2124	0.0414	0.2508	0.3617	0.3711	0.3884	0.1708	0.1787	0.2423	0.11

6. Experimental results

Experiments were performed with a known feeder that has already been analyzed in the literature by Huang [15] and Chiang and Jumeau [8]. This feeder was originally presented in [1] and is redrawn in Fig. 3 for the sake of clarity and completeness. In this figure, the italic numbers denote the labels of the distribution lines, and buses are identified by the number positioned inside of them.

The experiments were divided into cases 1, 2, and 3. Case 1 allows the original real and reactive power demands values in [1] to vary within a tolerance of 8%. Fig. 4(a and b) shows the ranges of each bus. Case 2 considers distinct perturbations to the load values on each bus. Fig. 5(a and b) shows the ranges of the load values of each bus. Case 3 uses the demand curves obtained in [28] to describe seasonal variations in loads according to five profiles. These profiles were assigned to each load bus of the original test feeder in [1] according to the range of the value loads. Uncertainties were incorporated by assigning specified ranges to each bus by considering the minimal and peak values ($\equiv$ nominal values of the original test feeder) of each profile (Fig. 6(a and b)). It is noteworthy that the uncertainties in cases 2 and 3 are more significant than in case 1 and that they are related to considerable oscillations in the load demand.

Computational experiments were performed using Matlab version 7.10.0.499 (R2010a) and the INTLAB toolbox [24], running on a i3 Core CPU with 2.13 GHz and 4 GB of RAM memory, to evaluate the simulations involving interval analysis. The following evolutionary algorithm parameters were held fixed: population size (60), maximum number of generations (30), and probabilities of crossover (0.98) and mutation (0.05).

6.1. Case 1

In Table 1, the first and second data columns present the results obtained in [15] and from the MOEA-DFR, respectively, for an uncertainty-free 70-busbar feeder problem. The two sets of results are in agreement, which validates the MOEA-DFR in the uncertainty-free case. The initial configuration was assumed to be the same as that used in [15]. It is important to note that even though the proposed formulation in [15] was multi-objective, the fitness assignment was performed by a weighted-sum of objectives, and the authors provided only a single solution rather than a Pareto frontier. The MOEA-DFR generates a frontier, and we have shown that the Huang solution is contained on the achieved frontier. The last two data columns in Table 1 show the ranges of each objective of the Huang solution and one solution pertaining to

the best frontier achieved using IMOEA-DFR, respectively, when uncertainties are included in the model. In this case, the two solutions are non-dominated by each other under Definition 6 (Section 4). Therefore, given the uncertainties in case 1, both solutions can be properly adopted in real-world environments. In contrast to other methods, the IMOEA-DFR accounts for uncertainties in the modeling, and the robustness of the solutions is thus guaranteed. It is valuable to mention that the MOEA-DFR attained the Huang solution for ten distinct executions.

6.2. Case 2

Assuming the same uncertainty for all loads is naive because each bus generally has a distinct demand profile. Hence, in case 2, we assumed different levels of uncertainty for each load values. Fig. 5 presents the power demand range at each load bus. In case 2, we worked with three scenarios: (i) the original scenario described in [15], (ii) a perturbed scenario in which the uncertainties lead the load values to the boundary opposite that they were in original scenario when considering the power demand range in Fig. 5, and (iii) the interval scenario represented by the ranges at each bus in Fig. 5. Scenarios (i) and (ii) were solved by MOEA-DFR, while scenario (iii) was solved with the IMOEA-DFR. The frontiers obtained in the experiments in case 2 were built from ten distinct executions of the MOEA-DFR and IMOEA-DFR, whose parameters were adjusted to default values, as previously explained. The final frontier of each scenario was built by joining all non-dominated solutions belonging to the optimal frontier in each execution. Table 2 presents the distributions of the solutions found in each simulated scenario according to the number of switching operations. The MOEA-DFR found frontiers Fr_o and Fr_p , which have 40 and 65 feasible solutions, respectively, to the simulated 70-busbar system in original and perturbed scenarios. The intersection between Fr_o and Fr_p contained only the initial solution because the fourth objective of the formulation requires that this solution remains on the final frontier of both sets (see Section 3). Thus, we conclude that all solutions in Fr_o are unfeasible or suboptimal configurations in the perturbed scenario. To obtain configurations that are suitable in all combinations of the original and perturbed scenarios, we use IMOEA-DFR to solve the interval scenario, which considers sufficiently large uncertainty margins to encompass the original and perturbed possibilities. IMOEA-DFR found frontier Fr_I with 75 feasible solutions, of which fourteen solutions were shared with Fr_o and four solutions with Fr_p . As expected, only the initial solution exists at the intersection between frontiers Fr_o , Fr_p and Fr_I .

It is worthwhile to note that the Huang solution is not included in Fr_P and Fr_r . Thus, it is an unfeasible or suboptimal solution under the worst case uncertainties in case 2. Table 3 shows that it is sub-optimal in the perturbed scenario, consequently it still remains feasible. However, we cannot guarantee suboptimality or unfeasibility for other combinations of uncertainties within the range described in Fig. 5. In contrast, the solutions provided by IMOEA-DFR are assured to be robust in the worst cases of all uncertainties envisaged in the model.

6.3. Case 3

In this case, we assume that the nominal loads specified in [15] behave as described in [28] by using 24-h demand curves. Each of the six classes was delimited by intervals for classification purposes. Load values below 5 kVA are classified as urban lighting. The rural, urban residential, commercial, and light industrial load intervals are 5–15 kVA, 15–100 kVA, 100–400 kVA, and 400–800 kVA, respectively. Apparent powers larger than 800 kVA are associated with heavy industrial loads. The IMOEA-DRF solution, illustrated in Table 3, is compared to the Huang and original configurations (switches 70, 71, 72, 73 and 74 open) at seven instants in time. Fig. 7 clearly shows that the IMOEA-DRF solution outperforms the other solutions.

Table 4 provides more details about the performance of the different configurations. The cumulative power losses for each configuration are 524.2452 (original configuration – 100%), 291.6970 (Huang – 55.64%), and 273.6005 (solution proposed by IMOEA-DRF – 52.19%). Although the number of switching operations is included in our multi-objective formulation as in many other published works, a more distant configuration can be better in aggregate by preventing more maneuvers in subsequent situations. In this example, the IMOEA-DFR configuration is achieved using eight switching maneuvers, but there is no need to perform other reconfigurations due to load variations throughout the day. Given that switching maneuvers are avoided by supervisory centers unless they are critical due to the risks involved in such operations, stable configurations are desired.

Using the computational environment and the algorithm parameters previously described, IMOEA-DFR has spent an averaged runtime of 2294.02, 2271.67, and 2223.83 s for cases I, II, and III, respectively. Those averages were calculated for ten runs of the algorithm performed in each case. The overall mean runtime is around 2263 s (75 s/generation), almost 65 times greater than the time spent by the conventional MOEA-DFR. As expected, the interval analysis has caused an increase on the computing burden due to extra calculus made to an interval instead of a single value. However, the benefit comes from the identification of robust configuration proper to be employed in longer periods outperforming other configurations achieved by means of the conventional approach, as verified in Fig. 7.

Unlike mono-objective problems, solving multi-objective problems consists in finding a set of solutions. Indeed, the quality of such solutions is evaluated by metrics which reveal some key factors: set size, closeness of the solution set to the Pareto frontier, and distribution pattern over the n -dimensional space. In this work, the performance of the solutions is evidenced by the hypervolume (HV) and Inverse Generational Distance (IGD) metrics. HV calculates the volume of the region, in the objective space, which is covered by the non-dominated solutions. Table 5 presents the HV values obtained by IMOEA-DFR. Regarding HV metric, the algorithm obtained better results for cases I and III. Table 6 presents the mean values of IGD metric for each run of those three cases. This metric was taken in relation to the estimated Pareto frontier. Smaller IGDs correspond to better convergence of the evolutionary algorithm. As HV, the IGD metric indicated that IMOEA-DFR

obtained better results for cases I and III. Hence, the algorithm found more difficulty in case 2 (a scenario with high levels of uncertainty).

In summary, all of the experimental cases revealed that the non-robust solutions found by the MOEA-DFR perform similarly to the robust solutions achieved by the IMOEA-DFR whenever light perturbations are considered. However, the IMOEA-DFR outperformed the non-interval approach under severe load variations.

7. Conclusions

In this paper, the feeder reconfiguration problem in radial distribution networks was analyzed. We demonstrated that configurations obtained from uncertainty-free models may not perform as well as expected in the presence of uncertainties. Hence, load uncertainties must be taken into account to obtain a more realistic model that yields robust solutions, and the IMOEA-DFR is proposed to solve DFR in the occurrence of such uncertainties. Our results demonstrated that a configuration requiring more initial switching operations can be cumulatively better over long periods of time, as the aggregate number of switching maneuvers is minimized over time by preventing further reconfigurations necessary to handle load variations. In summary, our proposed method properly solves the DFR problem in uncertain power demand environment. The IMOEA-DFR successfully finds robust and reliable configurations in terms of safety, full compliance with power demand and constraints, which are each fundamental in practical situations.

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