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Possibilities of Dialogue between Physical Education and Mathematics through the Development of a Mathematical Modelling Practice

Possibilidades de Diálogo entre a Educação Física e a Matemática por meio do Desenvolvimento de uma Prática de Modelagem Matemática

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ABSTRACT

This article originated from a research developed in the Professional Master's Degree in Mathematics Education, at the Federal University of Ouro Preto, through which 2nd year high school students from a private school in Minas Gerais investigated the concepts of combined analysis and its possibilities in the analysis of the training prescription forms, aiming to minimize the problem of equipment congestion among the users of the bodybuilding modality, in gyms during peak hours. The purpose of this article is to present a possibility of dialogue between the fields of Physical Education and Mathematics mediated by a practice of Mathematical Modeling based on its socio-critical perspective. The methodological approach used was an adaptation of the Grounded Theory, which guided the researchers in the development of data collection instruments, as well as in data analysis and interpretation of the results obtained in this study. The participants used their mathematical knowledge to analyze and develop models for the prescription of training sheets, providing the development of critical and reflection on the practices adopted in gyms regarding training management and the quality of care provided by instructors, as well as on the role of Mathematics in other fields of knowledge.

KEYWORDS: Mathematical Modeling. Combinatorial Analysis. Training Prescription Sheets. Sociocritical Perspective.

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RESUMO

Este artigo se originou de uma pesquisa desenvolvida no Mestrado Profissional em Educação Matemática, da Universidade Federal de Ouro Preto, por meio da qual os alunos do 2º ano do Ensino Médio, de uma escola da rede privada em Minas Gerais, investigaram os conceitos de análise combinatória e as suas possibilidades na análise das fichas de prescrição de treino, visando minimizar o problema do congestionamento de aparelhos entre os frequentadores da modalidade de musculação, nas academias em horários de pico. O objetivo deste artigo é apresentar uma possibilidade de diálogo entre os campos da Educação Física e a Matemática mediado por uma prática de Modelagem Matemática fundamentada em sua perspectiva sociocrítica. A abordagem metodológica utilizada foi uma adaptação da Teoria Fundamentada nos Dados, que norteou os pesquisadores na elaboração dos instrumentos de coleta de dados, bem como na análise dos dados e na interpretação dos resultados obtidos nesse estudo. Os participantes utilizaram o seu conhecimento matemático para analisar e elaborar modelos de prescrição de fichas de treino, propiciando o desenvolvimento da criticidade e da reflexão sobre as práticas adotadas nas academias referentes à gestão dos treinos e à qualidade do atendimento oferecido pelos instrutores, bem como sobre o papel da Matemática em outros campos do conhecimento.

PALAVRAS-CHAV: Modelagem Matemática. Análise Combinatória. Fichas de Prescrição de Treino. Perspectiva Sociocrítica. Academias de Treino.

Introduction

This article presents an excerpt from a research developed in the Professional Master's Degree in Mathematics Education, at the Federal University of Ouro Preto, between 2018 and 2019, through which the contributions of the socio-critical perspective of Mathematical Modeling in the development of concepts of combinatorial analysis by 2nd year high school students in a private school in the eastern region of Minas Gerais.

Thus, the most important purpose of this article is to present and discuss a Mathematical Modeling practice that provided a possibility of dialogue between the fields of study of Physical Education and Mathematics, which was mediated by its socio-critical perspective. We emphasize that this investigation was also conducted outside the school, in academies, characterizing the interdisciplinary aspect of Mathematics with Physical Education.

So, the participants involved in this study used the mathematical knowledge of combinatorial analysis to propose solutions to the problem of congestion in gyms, taking a critical and reflective point of view on the quality of services offered by these spaces, as well as on access people to these services.

The motivating issue of the Mathematical Modeling work was the search for solutions to the problem of congestion in the weight room that is faced by those who attend these training sessions, in many gyms, often considering the need for equipment rotation among those who practice them.

In this sense, the participating students were invited to investigate how the training prescription process occurs, through visits to gyms, interviews with the instructors who prepare these forms, and also by observing how these training spaces are administered.

In addition, these participants explored possibilities for analyzing the training prescription forms using the knowledge of combinatorial analysis as support, in the search for a solution to the problem of equipment congestion.

For the development of this approach, some tasks developed in the blocks of activities that made up the field work of this research were analyzed. Thus, we will show some excerpts from the meeting with the participant physical education teacher, which preceded the activity of visiting the gyms, as well as the discussions of the students participating in this study during the presentation of the interviews carried out in the activity of visiting the gyms.

We will also present some excerpts from the analysis process of the training situations that were configured through the mathematical concepts of combinatorial analysis, through the elaboration of mathematical models, which were developed as a proposal to analyze, minimize and/or reduce the relay of devices during training in gyms.

Thus, in the next topics, we will describe the theoretical choices that supported this investigation in Mathematical Modeling, as well as the methodological design used in data analysis and interpretation of results, and we will also discuss information on how the activities proposed in the classroom were developed by these participants.

Mathematical modeling: a pedagogical action mediated by the socio-critical perspective

In general, the teaching and learning process in Mathematics has raised concerns for the school community, society and researchers in Mathematics Education. However, what we observed in the researches is that the Mathematics taught in schools and the pedagogical practices employed in the teaching and learning process of the contents of this discipline did not follow the social and technological evolution of the post-modern society (ROSA; OREY, 2012).

According to Barbosa (2004), for more than two decades, we discussed the inclusion of Mathematical Modeling works in the school curriculum, mainly in Basic Education. Generally, five arguments are presented to defend the insertion of Modeling in the teaching and learning process in Mathematics, which are related to:

motivation, facilitation of learning, preparation to use Mathematics in different areas of knowledge, development of exploration skills and understanding of the sociocultural role of Mathematics in society.

In this sense, Rosa (2010) states that the motivation for using Modeling as a strategy in the teaching and learning process of Mathematics is to lead students to understand mathematical concepts through contextualization, so that they can develop a pre- - willingness to learn mathematical content by understanding its value in solving problem-situations faced daily.

Therefore, the main objective of pedagogical practices based on Mathematical Modeling is related to the critical development of students stimulated by reflective discussions among peers during the development and conduction of the Modeling process in the pedagogical action triggered in classrooms (ROSA; OREY, 2007).

Thus, the motivation for the socio-critical perspective of Mathematical Modeling is based on the proof that Modeling activities can start from the use of everyday problem situations, enabling the discussion of the nature and role of mathematical knowledge in the understanding and elaboration of models. mathematicians in society (ALMEIDA; SILVA, 2010).

The use of the sociocritical expression, as defined by Barbosa (2006), is related to a critical and reflective rereading of the role that Mathematical Modeling plays in the contemporary world. This context enables the manifestation of themes related to the role of Mathematics in society, through the elaboration of activities proposed in the classroom through which students establish a critical look at reality and the problems that surround them. In this approach, students resort to mathematical knowledge in search of solutions to minimize current conflicts and/or interpret them in the context in which they are inserted.

For example, Jacobini and Wodewotzki (2007) emphasize that the work with Modeling must be directed towards the political and social growth of students. Thus, from discussions held in Mathematics classes about problem situations of a cultural, social, economic, political and environmental nature, which result from the activities carried out daily; there is a need for teachers to encourage the active participation of students in the study of these issues, seeking to deepen critical reflections on investigations carried out in their own sociocultural context.

In this way, Mathematical Modeling activities, from a socio-critical perspective, make the classroom a learning environment in which everyone involved in this process participates equally in mathematical learning. This environment reveals the use of mathematical thoughts and reasoning that respect contrary ideas, as they aimed to lead the participants in the observation of how Mathematical Modeling and models are used for the analysis of proposed problem situations, enabling decision-making regarding these investigations (SILVA; KATO; PAULO, 2012).

However, before proposing an experience of Mathematical Modeling in the classroom, Barbosa (2004) emphasizes that it is essential to have clarity on the role of teachers and students in the development of activities applied in this pedagogical action. Thus, to guide teachers in planning actions that can be developed during the Modeling experience, 3 (three) Mathematical Modeling Cases are suggested, which differ in relation to the role that teachers and students play during the performance of these tasks.

In Case 1, the teachers present to the students a certain problem-situation with the quantitative and qualitative data necessary for the search for an answer to the problem presented in the classroom. In this experience, “students do not need to leave the classroom to collect new data and the activity is not very extensive” (BARBOSA, 2004, p. 5).

In Case 2, the teachers present a problem situation for the students to investigate, however, the necessary data for the analysis and the search for an answer to the problem proposed in the classroom are not provided. In this context, students are faced with only the problem to investigate, but they have to leave the classroom to collect the data and information necessary to conduct the investigation. Teachers are only responsible for the task of helping students to formulate the initial problem (BARBOSA, 2004).

In Case 3, teachers propose a theme or ask students to choose the theme they want to investigate. In this configuration, the experience of Modeling in the classroom is related to the proposition of projects developed from themes considered as non-mathematical. In this approach, the formulation of the investigation problem, the collection of data and the resolution of the problem-situations proposed in the classroom are tasks performed by the students (BARBOSA, 2004).

In this scenario, Rosa and Orey (2007) argue that it is through the interaction of teachers with students that the teaching and learning process in Mathematics is provoked and constituted through Modeling. However, the learning of mathematical content starts according to the students' purpose, as each one of them has a predisposition to act, react, reflect and modify the environment in which they live, systematically transforming it, aiming to seek the common good of society.

Methodological procedures

In the bibliographical research carried out for the development of the theoretical foundation of this study, between the months of November 2020 and January 2021, in the theses bank of the Coordination for the Improvement of Higher Education Personnel (CAPES), we looked for abstracts of dissertations and theses that discussed the proposed theme for this investigation using keywords, such as Mathematical Modeling, Combinatorial Analysis, Mathematics Education and Physical Education.

In this search, we found the occurrence of only one research, Martins (2009), whose results show the development of an interdisciplinary dialogue in a pedagogical practice in Early Childhood Education. However, the results obtained in this study are disconnected from the problem related to the research proposed in this article, showing the need to deepen this theme.

In this context, this research was conducted through a qualitative approach, as the methodological procedures used in this study were related to the collection and production of data through the interactive, cooperative and collaborative contact of the teacher-researcher with the students participating in this investigation.

We emphasize that the name professor-researcher was used in this research to refer to the first author of this study who was also the professor of the Mathematics discipline of the researched class. It is important to emphasize that the professors-researchers consider the teaching practice as a means, a foundation and a destination of knowledge and actions that raise a critical and reflective action on the practice itself (MIRANDA, 2006).

In this study there was the participation of 17 students from the 2nd year of high school, from a private school in the eastern region of the state of Minas Gerais, as well as a Physical Education teacher, who was invited to provide participants with technical elements related to the process of preparation of training prescription forms for gyms.

To ensure confidentiality regarding the identification of participants in this study, these students were identified with odd and even numbers adjacent to the letters M and F, for males and females, respectively, such as, for example, M for males with odd-numbered (M1) and F for the even-numbered female participants (F8).

An adaptation of Grounded Theory (GLASER; STRAUSS, 1967) was used as a methodological approach in this study, as there was neither the elaboration of the selective category nor the writing of an emerging theory. Therefore, this theory was used in order to seek an answer to the research question through data analysis and interpretation of the results obtained during the fieldwork of this research.

In this study, the use of theoretical assumptions of Grounded Theory was used from the development of instruments used in data collection to the process of data analysis and interpretation of the results obtained in this investigation. In the elaboration of the data collection instruments, we chose to use open questions instead of closed ones in order to create opportunities for the active participation of students to express their reasoning and conclusions.

In the process of analyzing the data and interpreting the results, the methodological procedures adopted made it possible, through the conduct of open and axial coding processes, to identify the contributions of the socio-critical perspective of Modeling during the analytical and interpretive processes of this investigation.

In this methodological design, the results were interpreted in a systematic way, as it aimed to enable the production of a methodological model theoretically based on the data. This methodology expanded, during the development of the analytical and interpretive phases of this research, the constant interaction between data collection, analysis and interpretation of the results obtained in this study (STRAUSS; CORBIN, 1990).

The codings shown in tables 4 and 5 are related to the open and axial coding process, which was used in the analysis of the data collection instruments proposed for this study. In this process, the preliminary codes were subtitles used by the professor-researcher in the data analysis, as it aimed to establish symbolic units for grouping these codes by conceptual similarity, enabling the elaboration of conceptual categories (MILLES; HUBERMAN, 1994).

In this study, the open coding process was performed manually by reading and recording the information contained in the data, enabling the identification of preliminary codes. Then, using axial coding, the preliminary codes were reorganized into a higher level of conceptual abstraction, which gave rise to the conceptual categories (BASTOS, 2019).

Continuing with this methodological approach, this research was inspired by Case 2 proposed by Barbosa (2004), in which teachers present a problem situation for students to investigate. Thus, in this study, the participants collected data outside the school environment during the visit to the gyms.

After this visit, the students participating in this study analyzed the problem-situation related to congestion in gyms at certain hours of operation, which are called peak hours and which generally occur from 7:00 pm to 9:00 pm, during which, almost always, there is a need to rotate equipment in the weight room.

So, for the development of the Mathematical Modeling practice proposed in this research, the students visited the academies they chose, whose objective was related to the investigation of combinatorial analysis concepts, such as, for example, permutations and combinations, which could be present in the elaboration training prescription sheets, which are designed to guide gym students during their training.

Thus, for the data collection process, 4 (four) blocks of activities were applied that made up the documental record of this study, as well as the field diary used by the professor-researcher. It is noteworthy that the blocks of activities were structured according to the steps of Mathematical Modeling proposed by Rosa (2005), which are composed of 3 (three) distinct phases, which enable the organization of this process, as this structuring aims at a better use of the pedagogical practice of Mathematical Modeling in classrooms.

In this structure, Mathematical Modeling comprises 3 (three) phases: a) Initial phase that refers to the preparation of the Modeling, b) Intermediate phase that comprises the development of Modeling and the elaboration of mat models and c) Final phase that relates to the presentation of the Modeling projects and the delivery of the final report. Table 1 shows the blocks of activities proposed for document registration.

Table: 1: Blocks of activities proposed for document registration

BLOCK 1: INITIAL PHASE - MODELING PREPARATION **Checking tacit knowledge[3]**	
Activity	**Objective**
Division of participating students into groups of 06 (six) components to carry out individual activities in the classroom. Systematization of combinatorial analysis concepts. Research on combinatorial analysis concepts.	Verify the use of combinatorial thinking acquired by the participating students in their daily experiences with this mathematical content. Develop with the participating students the systematization of the concepts of combinatorial analysis.
BLOCk 2: INTERMEDIATE PHASE - MODELING DEVELOPMENT	

[3] Tacit knowledge is impregnated in the personal experience of individuals, being acquired and accumulated through of individual experience, involving impalpable factors such as beliefs, perspectives, perceptions, systems values, ideas, emotions, norms, hunches and intuitions. Tacit mathematical knowledge relates to the ways in which students use mathematical concepts and appropriate experiences mathematics experienced in their daily life through their everyday experiences (ROSA; OREY, 2012).

Visit to the city's gyms	
Activity	**Objective**
Meeting with high school Physical Education teachers and with the Physical Education teacher participating in this study. Preparation of the tour guide to the city's gyms to obtain information on the preparation of training prescription forms. Visit to gyms.	To investigate whether the participating students perceived the presence of mathematical knowledge related to Combinatorial Analysis in the process of preparing training prescription forms and their functioning in gyms.
BLOCK 3: INTERMEDIATE PHASE - MODELING DEVELOPMENT Analysis of training prescription sheets	
Activity	**Objective**
Analysis of data collected in surveys conducted on training prescription forms. Mathematical investigations of the information obtained. Elaboration of models based on training prescription sheets. Production of short videos.	Discuss in group, with the mediation of the teacher-researcher, the possible connection between the concepts of combinatorial analysis with the process of elaboration of training prescription forms. Develop mathematical models.
BLOCK 4: FINAL PHASE - PRESENTATION OF THE MODELING AND DELIVERY OF THE FINAL REPORT Presentation of results and evaluation of the Modeling process	
Activity	**Objective**
Elaboration of the final report on the modeling activities related to the combinatorial analysis contents. Presentation of the models and the results obtained in the researches carried out. Delivery of the final Mathematical Modeling report.	Discuss the preparation, presentation and delivery of the final Mathematical Modeling report. Presentation of the final report and the mathematical models developed by the groups of students. Evaluation of the presentation and report by the board members[4].

Source: Adapted from Bastos (2019, p. 125)

In this context, starting from this reality, the students investigated how the process of elaboration of training prescription forms takes place in gyms. In this sense, these participants suggested solutions for the problem-situations found in the gyms, aiming to analyze them, taking as inspiration the knowledge of combinatorial analysis for the prescription of these trainings.

Therefore, the students participating in this study investigated how the training prescription process occurs in order to explore the possibilities of its configuration through knowledge of combinatorial analysis in order to find solutions to the problems found in these gyms.

Introducing and discussing modeling tasks

In the second block of activities, a task was proposed in which students visited the city's gyms seeking information about the process of preparing training prescription forms. The main objective of this activity was to investigate whether these participants would identify the presence of combinatorial concepts in the

[4] The teacher-researcher, the high school pedagogical supervisor, 2 (two) high school teachers, the Physical Education teacher participating in this study and an instructor from one of the visited gyms participated of this meeting when they compose the evaluation panel of the final reports.

making of these forms, as well as their functioning during training hours in the weight room.

However, before the participants developed the activity of visiting the gyms, the students were guided by the Physical Education teachers of the school and also by the Physical Education teacher who participated in this study, with reference to the protocols that guide Education professionals Physics during the process of prescribing training sheets. According to Gentil (2005), these protocols are related to the variables considered in these forms, such as, for example, the intensity, volume, weekly frequency of training and the progression of the exercises that will be proposed in these protocols.

After this meeting, which was held in a Physical Education class, the participating students gathered in a group to prepare the interview script that should be used to guide them in visiting the gyms. This activity was marked by the role of 3 (three) participating students, who were members of each of the 3 (three) groups. However, this fact did not hinder the participation and contribution of other students in this activity. Table 2 shows an excerpt of the dialogue between the participating students and the teacher-researcher during the elaboration of this script.

Table 2: Excerpt from the dialogue between the professor-researcher and some participants for the collective construction of the interview script for the visit to the gyms

Professor-Researcher: I think this question, do you agree to join it with this one? How does the initial assessment take place when a person looks for a gym? What issues are raised and taken into account in the evaluation? M13, M15, F4, F14, M1: No [answered together]. F4: No, because in this evaluation they consider that thing that puts its foot on and goes like this, it considers its elasticity, that kind of thing. M13: No, they look at it's weight. F14: This question should not be asked, because not all academies do evaluations. I didn't do it myself at the gym I worked out. It's because there are people who don't. There are people who don't pay to be able to do it. M15: What is intensity and what is charge? F4: How are variables considered? If the guy quotes, oh, intensity is considered, then you ask: “What do you specifically consider as intensity?”. M7: I think you have to ask how do you avoid congestion? M15: I thought of asking: “If they take into account the time the person goes to the gym, and how many people will be in the gym at that time, on average?

F4: And more, there is the rotation of people. How the rotation of people is organized.

M15: What if the guy says he has no strategy to minimize this?

F4: I think you also don't have to ask the question the way it is there. This is just a script to help. You have to be attentive in the conversation. I think you have to ask how many instructors are needed for peak hours. How many instructors do you have on average and how is this monitoring?

M15: How does the gym work during peak hours? Is there congestion? Is there a relay of devices?

M17: Question also how do you avoid it?

F4: I think you have to ask what is the maximum capacity per hour.

Source: Adapted from Bastos (2019, p. 224)

The next stage of the field work of this study was related to the visit to the academies, with the professor-researcher accompanying each of the groups in carrying out this activity. Students recorded, through audio recording, the interview conducted with the instructors at the gym chosen for the visit.

After carrying out the visits, the members of each group transcribed these interviews, sharing them with the other students participating in discussions held in the classroom. It is important to clarify that, for these presentations, three class times of 50 minutes each were necessary.

In this sense, Rosa (2005) emphasizes the importance of teachers organizing visits to institutions, as well as interviews and lectures with professionals who work in areas related to the chosen topics, as this approach aims to guide students in understanding the topic, helping them in the process of formulating, solving and analyzing mathematical models.

After carrying out this activity, the professor-researcher proposed a critical and reflective reflection based on the experiences lived by the participants in the visits to the gyms, discussing issues such as, for example, the strategies used by the gyms to avoid the problem of congestion, the quality of services offered by these companies, the health benefits of physical activity and the population's access to this type of training in general.

The notes recorded in the professor-researcher's field diary show that this discussion was marked by an active participation of students, which signaled the predominance of inequalities in the quality and management of training between small, medium and large academies.

In this direction, Bastos and Rosa (2020) commented that participant M13 argued that “there they think of everything to avoid congestion. When you enter the middle there is the reception, on the right side are the aerobic exercises and on the left are the weight training exercises" while participant F4 criticized the service offered by the gyms by stating that:

> I see that today most of these academies are interested in having more students. Then, I don't know if exercising in a gym will be so good for your health. I don't know if the instructor will prescribe a correct workout or if he's just interested in my money, got it? That's what I think!

Thus, in this study, we intend, through Mathematical Modeling activities, to establish a democratic space in the classroom, with critical and reflective dialogues between the teacher-researcher and the students, as it is important that teachers assume a relevant role in the political formation of students (ARAÚJO, 2009).

When sharing with her colleagues a personal experience witnessed in one of the gyms visited, participant F4 stated that “I worked out for a year and the instructor barely knew my name. If I missed training, the instructor wouldn't miss it. There were too many students for the small number of instructors at night” (BASTOS; ROSA, 2020).

Regarding the health benefits of physical activities, some issues were discussed by the participating students, such as commitment and attendance in the practice of these activities. Table 3 shows an excerpt of the dialogue between four participants in this study about this problem.

Table 3: Excerpt from the dialogue between participating students about the importance of physical activity

M5: I think it depends a lot. Because, for example, like [participant M15] said, it is variable. Let's suppose, I need to go to the gym, but during the week I have some obligation and then I only go one day and I'm missing two. So, I get there, do half the exercises and leave. This exercise will not be good for my health. F20: Yeah, but that's your problem! You have to organize yourself to do everything right. M13: It was like he [the interviewee] said at our academy, you have to do your part. For you to see results you have to train properly. So, in general, the gym is good for your health. In this case that you [addressing participant M5] said, it will depend on the students doing the training properly so that it is good for their health. F12: It was useless for a person to be going to the gym with a personal trainer if they are not doing their part, such as eating right, drinking water and sleeping. M13: That's what he said [addressing participant F12], the personal trainer isn't going to take

the amount of water for you, he's not going to eat the right amount you need.

Source: Bastos (2019, p. 247)

When being discussed about equal access to services provided by academies, participating students signaled about the inequalities of admission in establishments that were visited by participants in their groups. For example, participant F4 shared that "I don't think everyone has access to the gyms they want. Just like we heard in group presentations. The most organized thing we saw was the gym visited by one of the groups, which is the most expensive". Confirming the issue of inequalities in access to services provided by training academies, participant F4 reflected that:

> I think that if we think that the gym is important for health, accessibility for everyone should be thought of. A person who earns a minimum wage is unable to pay even a monthly fee of eighty reais, let alone a gym that covers between one hundred and two hundred reais.

Still discussing access to the gyms, participant M13 justified to the other students that the "high price of the monthly fee at the gym we visit is to be able to maintain the quality of services and exchange the equipment they use every three months". Therefore, the notes recorded in the professor-researcher's field diary show that, in this participant's understanding, the high cost of tuition is valid when there is proof of the quality of services offered by the academies.

In response to student participant M13, student participant F14 disagreed with her argument stating that "my brother lived in the United States and told me he paid 10 dollars a month to train at a large gym". However, 3 (three) participants responded that "we cannot compare the economic, social and cultural reality of the Brazilian population with the North American one". In this sense, participant F12 stated that "it is true, as we cannot compare with the United States, as we are in Brazil".

In this direction, Rosa and Orey (2012) state that it is necessary for Mathematics teachers to provide experiences in the classroom with the use of contexts other than school ones so that these professionals enable the development of students' logical reasoning, in a critical and reflective way. with respect to the results of mathematical models contextualized in everyday life.

After completing this task, the participants performed in the classroom, the third block of activities, whose objective was to connect the development of the tasks performed in the first block of the documentary record[5] with the activities of analysis

[5] In this block of activities, two tasks were proposed involving each combinatorial concept, starting from a more concrete conceptualization, so that they could solve them without the application of

and preparation of training prescription forms, as well as its relationship with mathematical knowledge regarding the contents of combinatorial analysis and the development of mathematical models related to this theme.

Later, the teacher-researcher started the third block of activities by asking the participants to read the text provided, as it contained information about the training modality discussed according to the knowledge of a Physical Education teacher. Then, the teacher-researcher mediated a discussion with these participants about the importance of circuit training, asking students to solve the proposed questions for their resolution in the classroom.

During the analysis of these activities, Bastos and Rosa (2020) proposed that the participating students make training prescription forms. For example, 16 participating students correctly answered that there are 120 ways to determine, through the use of the permutation concept, how many ways the gyms visited could organize a training circuit with 5 (five) people.

Thus, participant M13 commented that this problem-situation “is a permutation because you are going to make an exchange”. In this direction, these participants mathematized this problem-situation by elaborating a mathematical model through the elaboration of the mathematical model related to the permutation formula: P5=5!= 5x4x3x2x1=120.

However, 11 participating students had difficulties in interpreting the problem-situation statement through which they would have to determine the maximum number of clients who attend weekly the training circuit modality to consider the possibility of maximum capacity in all sessions proposed by the academy.

Data analysis shows that the 17 participants critically analyzed this situation by proposing alternatives to solve it according to the information given in this problem-situation so that mathematical knowledge was not disconnected from their daily and school experiences (BASTOS; ROSA, 2020).

In this context, participant F4 stated that it was necessary to "determine the maximum capacity, to know how many people there were at most in all training sessions" while participant M15 reflected that "when determining the maximum capacity in this training, it should be considered that a person will at the gym to do circuit training twice a week”.

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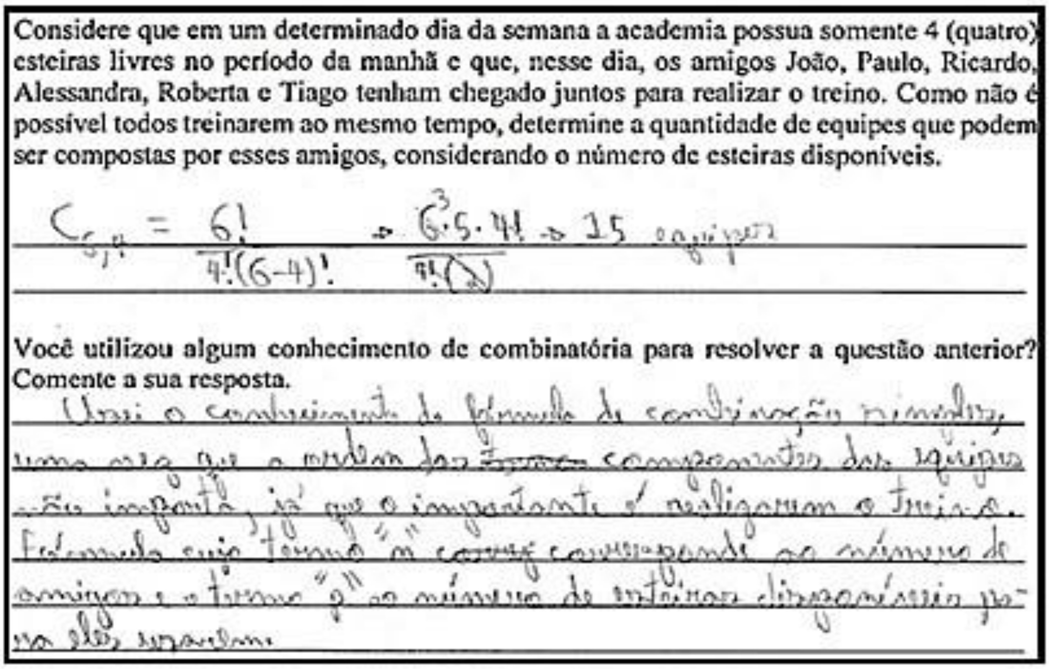
formulas. At the end of this block, the teacher-researcher proposed to the participating students to carry out a survey on the meaning of combinatorial analysis.

Thus, these participants discussed the possible results of this issue by also verifying the possibility of ambiguity in this statement[6] through critical and reflective discussions that they developed on this issue (BASTOS; ROSA, 2020).

Then, the teacher-researcher proposed the following question: Consider that on a given day of the week the gym has only 4 (four) free treadmills in the morning and that, on that day, friends João, Paulo, Ricardo, Alessandra, Roberta and Tiago have arrived together to perform the training. As it is not possible for everyone to train at the same time, determine the amount of teams that can be composed of these friends, considering the number of mats available. Did you use any knowledge of combinatorics to solve the previous question? Comment on your answer.

The analysis of the answers given by the participating students to this question shows that 16 participants correctly solved it through the elaboration of a mathematical model that represented this problem-situation. Figure 1 shows how participant M15 correctly solved this problem situation, representing it through the use of a diagram.

Figure 1: Mathematical model developed by the participating student M7


Considere que em um determinado dia da semana a academia possua somente 4 (quatro) esteiras livres no período da manhã e que, nesse dia, os amigos João, Paulo, Ricardo, Alessandra, Roberta e Tiago tenham chegado juntos para realizar o treino. Como não é possível todos treinarem ao mesmo tempo, determine a quantidade de equipes que podem ser compostas por esses amigos, considerando o número de esteiras disponíveis.

$C_{6,4} = \frac{6!}{4!(6-4)!} \rightarrow \frac{6^{3} \cdot 5 \cdot 4!}{4!(2)} \rightarrow 15$ equipes

Você utilizou algum conhecimento de combinatória para resolver a questão anterior? Comente a sua resposta.

Usei o conhecimento da fórmula de combinação simples, uma vez que a ordem dos componentes das equipes não importa, já que o importante é realizarem o treino. Fórmula cujo termo "n" corresponde ao número de amigos e o termo "p" ao número de esteiras disponíveis para eles usarem.

Source: Bastos (2019, p. 282)

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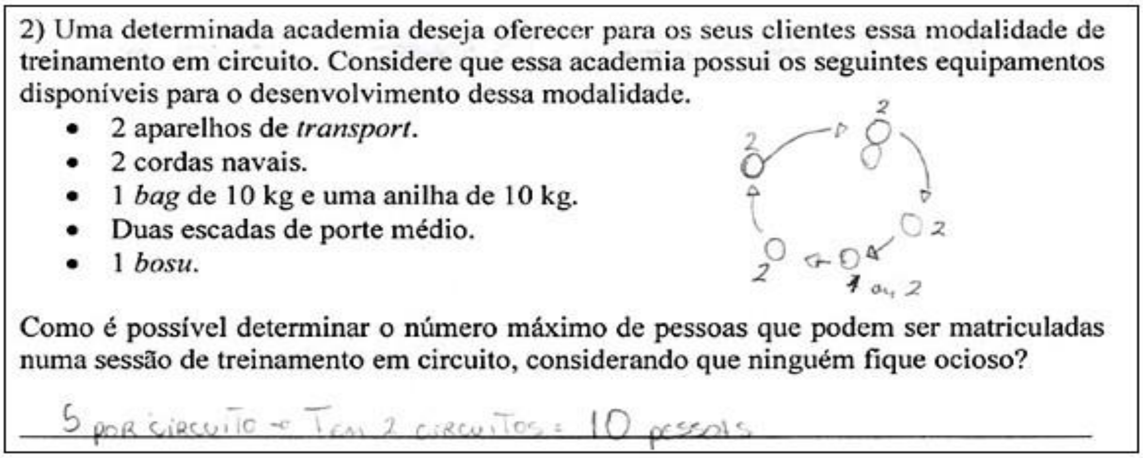
[6] The owner of this gym offers this service to its customers in monthly packages, with two workouts weekly, which will take place from 8 am to 9 am or from 4 pm to 5 pm, on Monday and Friday of every week. Determine the maximum number of people who will attend this modality per week. Consider the possibility of maximum capacity in all sessions.

On the other hand, 1 (one) participating student, M9[7], answered this question incorrectly, as he stated that the "number of teams that can be formed by these friends is four". Thus, the notes recorded in the professor-researcher's field diary show that, when explaining his resolution, this participant replied that he did not use the concept of combinatorics, as only "I used the reasoning of using two people for each mat". This approach demonstrated an interpretative gap of the given problem-situation in relation to the mathematization procedure used in its resolution.

Continuing with the analysis of the preparation of the prescriptions of the training sheets carried out by the participants, it can be inferred about the presence of mathematization processes and the elaboration of mathematical models in the resolutions presented by these students to determine the maximum number of clients that could be enrolled in a circuit training session, disregarding the idleness of the devices (BASTOS; ROSA, 2020). Figure 2 shows how participant M15 correctly solved this problem situation, representing it through the use of a diagram.

Figure 2: Mathematization process used by the participating student M15


2) Uma determinada academia deseja oferecer para os seus clientes essa modalidade de treinamento em circuito. Considere que essa academia possui os seguintes equipamentos disponíveis para o desenvolvimento dessa modalidade.

- 2 aparelhos de *transport*.
- 2 cordas navais.
- 1 *bag* de 10 kg e uma anilha de 10 kg.
- Duas escadas de porte médio.
- 1 *bosu*.

Como é possível determinar o número máximo de pessoas que podem ser matriculadas numa sessão de treinamento em circuito, considerando que ninguém fique ocioso?

5 por circuito → Tem 2 circuitos = 10 pessoas

Source: Bastos (2019, p.350)

For example, for Bastos (2019), the answers given by participating students to the question: Consider that a certain gym has an audience of 200 people working out in the weight room, with the presence of 5 (five) instructors to assist students in the room of bodybuilding, in a certain period. Taking as a reference the number of instructors at this academy and the average number of students each of them attends, do you believe it is possible to guarantee good service at this academy?

[7] This participant has a cognitive deficit, according to the medical report presented by their guardians. So, it was prepared by the pedagogical team of the school, in which this study was carried out, together with the teachers in the exact sciences, an Individual Development Plan (PDI) for this student, for presenting limitations in the exact area.

Comment, they show that, when responding to this problem-situation, the 17 participating students commented on the impossibility of offering a good quality service to the clients of this academy.

In this sense, according to the information obtained during the visit to the academy, participant M3 replied that:

> (...) with this average number of students served by each instructor in this academy, I believe that it will not be possible to guarantee good service to students since, based on the interview we conducted at the academy, we came to the conclusion that each instructor is able to meet a maximum of 20 students and, in this academy, the average is 40.

In accordance with this assertion, the notes recorded in the teacher-researcher's field diary show that the discussions held with the participants of this study during the performance of the proposed activities in the classroom enabled the students to develop a critical and reflective posture on this issue. when discussing the actions that could be taken by gyms to reduce and/or minimize the problem of congestion of equipment available for the use of their customers (BASTOS, 2019). In this direction, these:

> (...) actions were related to: a) equipment rotation so that all clients could work out, b) changes in the students' training prescription form, c) equipment change and reduction in the number of repetitions, d) performance other exercises while customers wait for the equipment to vacate, e) increase in the amount of equipment in the gyms, f) work with circuit training, g) establishment of a limit of customers in the gyms and h) the quality of service offered by the gyms (BASTOS ; ROSA, 2020, p. 9-10).

In this way, the socio-critical perspective of Mathematical Modeling allowed the students participating in this study to trigger a discussion about the role that Mathematics assumes in society and, also, regarding the nature and relevance of the elaboration of mathematical models that aim to seek the understanding of the sociocultural environment in which students are inserted (ROSA; OREY, 2007), especially in relation to the academies that are environments outside the school.

Consequently, the analysis of the answers given in the data collection instruments showed that the participants in this study became aware of the importance of school mathematical knowledge that is used in daily activities practiced in gyms, as they were able to develop mathematical models for the prescription of training sheets that could foresee the possibilities to minimize the problem of congestion of equipment available to customers in these spaces (BASTOS; ROSA, 2020).

After carrying out this activity, 13 participants discussed what would be the necessary measures to propose an improvement in the operation of equipment rotation in gyms to avoid its congestion, as well as to reduce the waiting time of customers for its use.

So, these participants suggested that gyms use the circuit modality to minimize the problem of congestion in these devices. Thus, for participant F14, gyms should also "have a large amount of equipment so that there are no impediments related to overcrowding", in addition to the use of circuit training (BASTOS, 2019).

Continuing with the conduction of this analytical process, the professor-researcher used the assumptions of the Grounded Theory to elaborate Table 4, which shows the preliminary codes that were identified during the open coding process in relation to the analysis of the responses given by the participating students to activities related to the Analysis of Training Sheet Prescriptions by Participating Students.

Table 4: Preliminary codes collected in the Activity Analysis of Prescriptions of Training Sheets by Participating Students

Raw Data Collected	Open Encoding (Preliminary Codes)
If there are five people, four people will do the other training (24). But if there is more, it won't work (33). The order matters, he said here who is the first, which is the second (19). But, and this exercise here he said that it is one after the other (24). For this training to be carried out, there must be a certain order (19). It's a permutation. Because you are going to make an exchange (19). N factorial. P equal to 5 factorial. It will be 120 (40). P_5=5!=5·4·3·2·1=120 ways (41). Only 5 people can be enrolled in a session, taking into account that there are 5 exercises (19) and during all rounds they will be used (43). 1 group per circuit. 5 people = 6 g/hour = 30 people. 60 people a day. 2 hours a day (24). 120×6=720 6!=6·5·4·3·2·1=720 ways (41). Bike – 30s. Treadmill – 20s (24). Permutation: P5 (40). The amount of exercise is the same. 5!=5·4·3·2·1=120 (41). There, at the gym that we visited, there is also a rotation of equipment among students (33). Because there are more people than braces in gyms (33). In order not to spend money, most want to earn money (33). He is going to make a treadmill or a bicycle, he is not going to make a treadmill and a bicycle (19). It's because he says he's going to warm up on one of the devices, it's not like that three people want to do it on the treadmills, two want to do it on the bicycle, there's no such thing (33). There it is combination (19). We saw that in that activity of recording videos that the order of exercises matters in the order of execution (33). But it's	(19) Combinatorial thinking (reasoning) (24) Mathematizing problem situations (33) Critical-Reflective Thinking (40) Modeling problem situations (41) Making mathematical models

arrangement (19).	

Source: Adapted from Bastos (2019, p. 270)

After completing the determination of the preliminary codes in the open coding process of this activity, the professor-researcher proceeded with the axial coding to determine the conceptual categories. Table 5 shows the axial coding in relation to the analysis of the preliminary codes identified in the analysis of the activity: Analysis of Training Sheet Prescriptions by Participating Students, which helped the teacher-researcher in identifying the conceptual categories.

Table 5: Axial coding of the preliminary codes identified in the Activity Analysis of Prescriptions of Training Sheets by Participating Students for the identification of conceptual categories.

Open Encoding (Preliminary codes)	**Axial Coding**
(19) Combinatorial thinking (reasoning) (22) Resolution Strategies (24) Mathematizing problem situations (33) Critical-Reflective Thinking (40) Modeling problem situations (41) Making mathematical models	(Conceptual Categories)

Source: Adapted from Bastos (2019, p. 271)

This analytical process, which was based on the assumptions of Grounded Theory, enabled the development of a conceptual category called Mathematical Modeling Process, which helped the teacher-researcher in interpreting the results obtained in this block of activities.

In this direction, the interpretation of the answers given by the participating students to the questions proposed in the initial phase of modeling preparation, indicated a trend in the development of the Mathematical Modeling process, based on the recognition of the possibilities of applying mathematical knowledge in problem-situations faced in the daily life of academies.

In this context, it is noteworthy that one of the main characteristics of the Mathematical Modeling process is its use as a motivation for learning and the development of mathematical techniques, strategies and procedures that aim to assist students in solving everyday problems (BASSANEZI, 2016).

From this perspective, 15 students participating in this study used the mathematization process to correctly solve the problem-situations proposed in the classroom, demonstrating a concern with the critical understanding of these situations through reflection on the combinatorial strategies and procedures used in the development of mathematization of this process.

In this context, Rosa and Orey (2017) state that, in the process of mathematizing phenomena that occur in daily life, Mathematics is considered a human activity that includes the resolution of problems faced daily through analysis, reflection, explanation, the understanding and understanding of situations and phenomena present in everyday life.

According to the interpretation of the results obtained in this activity, it is inferred that these participants discussed critically when reflecting on the main aspects related to the preparation of prescription forms for weight training, as well as about the functioning of gyms and the role of instructors, connecting it with the problem situations experienced in this environment.

Consequently, Barbosa (2003) states that the inclusion of Modeling in the mathematical curriculum helps the use of Mathematics in different areas of knowledge, as students have the opportunity to develop the ability to apply it in various everyday situations, helping them to verify the importance of this science in everyday life.

Thus, these participants discussed the resolving processes of these situations, relating them to combinatorial concepts in the search for the development of mathematical models that would represent them through their translation into academic and/or school Mathematics language. This fact is corroborated by Rosa and Orey (2012) when they state that, in Modeling investigations, there is a need to use translation to describe the process of elaboration of models taken from reality, which can have a school/academic mathematical representation.

These participants also stated that knowledge of combinatorial analysis is present in gyms and in the organization of this training modality, highlighting the applicability of the permutation concept in the resolution of issues related to the circuit, aiming to minimize the problem of congestion in these environments. For example, student participant M7 replied that “the use of the circuit with the application of combinatorics would expand and optimize the use of the devices, not leaving people idle”.

This participant also commented that “the relationship of this training modality with combinatorial analysis is related to the use of permutation to discover circuit possibilities”. In this sense, for Rosa and Orey (2007), the socio-critical perspective of

Mathematical Modeling seeks to associate school mathematical knowledge with problem-situations presented in everyday life.

Thus, we infer that the socio-critical perspective of Mathematical Modeling contributed for the participants of this study to realize the importance of an active, cooperative, interactive and collaborative teaching process of learning combinatorial concepts that were related to daily activities practiced in the daily context of academies.

Similarly, the use of Mathematical Modeling in the classroom is configured by carrying out activities, through which students are encouraged to discuss Mathematics and its role in the context of everyday situations, other sciences and other fields of knowledge (BARBOSA, 2004).

Thus, the implication of this approach is related to the fact that students can get used to the development of a careful and holistic analysis of the problem situations proposed in the classroom and that are rooted in the performance of their daily activities (MORGADO, CARVALHO, CARVALHO; FERNANDEZ, 1991).

For example, this approach proposes the conduction of a pedagogical action that is disconnected from a mechanical and limiting process, which may make it impossible to apply combinatorial concepts in problem-situations present in daily life.

Final considerations

In this article, we chose to present an excerpt from one of the blocks of activities of a master's research through which it was proposed to carry out a Mathematical Modeling experience related to the development of combinatorial analysis concepts in a class of 2nd grade students. high school, in a private school in a city in the Rio Doce valley, in Minas Gerais, whose main objective was to show the possibility of developing a dialogical and interdisciplinary relationship between the fields of study of Physical Education and Mathematics.

In this sense, it is important to highlight the importance of the participating Physical Education teacher for this Modeling experience to be successful, as the sharing of specific academic knowledge of the training of this professional was essential for the participating students to obtain an initial understanding of what it is like. The prescription of the training sheets in the gyms was elaborated before the visit activity to these spaces was carried out.

We understand that this pedagogical action made possible the proximity between the university academic universe and basic education through the meeting in which the Physical Education teacher participating in this research, representative of the university education space, spoke with the students participating in this study.

Thus, the blocks of activities proposed in the field work of this study enabled the awakening of these participants' interest in carrying out the Mathematical Modeling activities proposed in the classroom. In this context, these activities provided the enrichment of mathematical knowledge through the contextualization of school mathematics in relation to the knowledge used by the participants of this study in their daily lives.

Thus, when analyzing the activities that make up these blocks, we infer that the students participating in this study used the concepts of combinatorial analysis in solving the proposed tasks by establishing their connection with the process of prescription of gym training, whose development was mediated by the use from the socio-critical perspective of Mathematical Modeling (BASTOS; OREY, 2020).

The notes recorded in the teacher-researcher's field diary showed that the discussions held on the possibilities of preparing training prescription sheets that sought to minimize the problems of equipment congestion in gyms triggered questions about the possible contributions of Mathematical Modeling to resolution of this problem situation. This pedagogical action enabled these participants to develop their critical and reflective sense to become aware of the proposition of solutions for the problem situations faced in the external environment of the gyms (BASTOS, 2019).

With regard to a proposal for a solution to the problem related to the congestion of customers on the gym equipment, Bastos (2019) states that one of the suggested suggestions was related to the adoption of circuit training practices by the gyms. Thus, participant M7 argued that:

> People also discovered that it is not good to have several people on different equipment in the gym at the same time. There better be a circuit. In the circuit you don't have idle people, everyone is working, so there is no congestion.

Consequently, the socio-critical perspective of Mathematical Modeling provided an awareness of the use of combinatorial concepts necessary for the preparation of training prescription forms, as well as the development of a critical analysis of the quality of services offered by these academies. So, this perspective contributed to the promotion of critical and reflective discussions about the role of mathematical models in gyms by proposing solutions for better use of equipment and training management from the implementation of the circuit modality, which aims to enable the decongestion of the devices used by customers in these training environments.

In this sense, Bastos and Rosa (2020) state the importance of stressing that discussions about the quality of care provided by instructors, the care to be taken when choosing a gym and the benefits of physical activity for health were also present in the discussions held during the development of the activities proposed in this experience carried out with Mathematical Modeling.

Finally, the search for innovative and differentiated methodological paths, which aim to integrate the daily lives of students with the mathematical practice in schools, enables the development of a link between the logical-formal structure of Mathematical Modeling and its use in classrooms, as it seeks understand and describe reality, providing students with the perception of having the central and active participation in the teaching and learning process of mathematical content in the school environment. Thus, we hope that this work can contribute to the development of Mathematics Education by proposing discussions on Mathematical Modeling practices through interdisciplinary dialogue with other fields of knowledge.

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